Distributed Amplifiers: A quick review

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Distributed Amplifiers: Why ?

1) Used in a few applications :

instrument front - ends,

wideband (military) receivers,

optical modulator drivers

2) Distribution concepts apply more broadly

phase - matching of traveling waves.

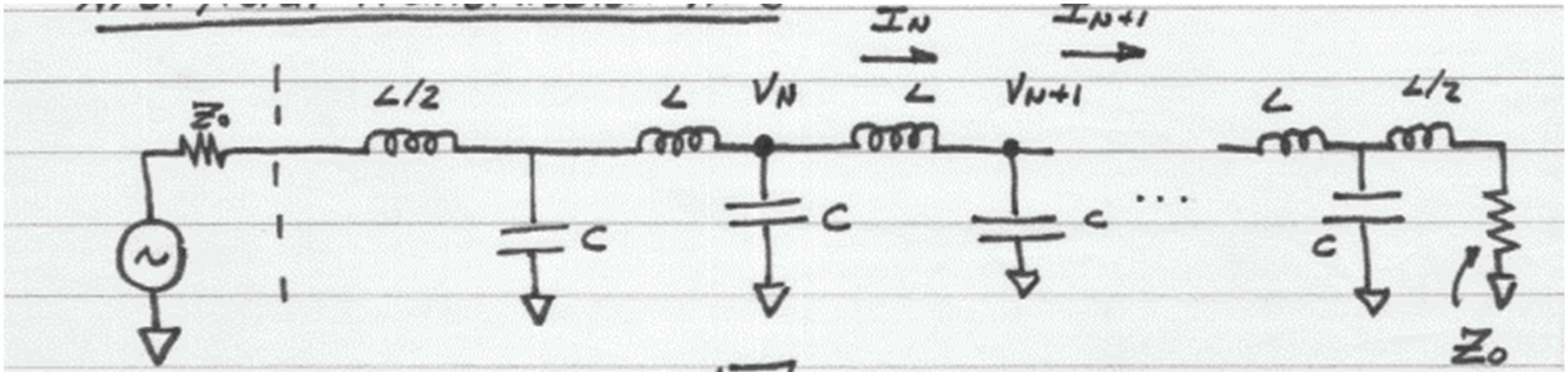
accumulated attenuation

synthetic transmission - lines.

#2 is our main motivation...

Also know as traveling - wave amplifiers

Synthetic (artificial) transmission-line



Design: $Z_0 = \sqrt{\frac{L}{C}}$ by choice.

To analyze: $V_{N+1} = e^{-\Gamma} V_N = e^{-A} e^{-jB} V_N$

$$\left.\begin{aligned} V_N(e^{-\Gamma}-1) &= -j\omega L I_N \\ I_N(e^{-\Gamma}-1) &= -j\omega C V_{N+1} \end{aligned}\right\} \text{solve for } e^{\Gamma}$$

$$\Rightarrow \cosh\Gamma = 1 - \frac{\omega^2 LC}{2}$$

define a cutoff (or Bragg) frequency

$$\omega_c = \frac{2}{\sqrt{LC}}$$

Synthetic transmission-line

<u>Then</u> $\cosh\Gamma = 1 - 2\omega^2/\omega_c^2$

but $\cosh\Gamma = \cosh(A + jB)$

$= \cosh(A)\cos(B) + j\sinh(A)\sin(B)$

<u>so</u>

$$\left(\cosh(A)\cos(B) + j\sinh(A)\sin(B) = 1 - 2\omega^2/\omega_c^2 \right)$$

examine how this behaves:

Synthetic transmission-line

$$\left(\cosh(A)\cos(B) + j\sinh(A)\sin(B) = 1 - 2\omega^2/\omega_c^2 \right)$$

examine how this behaves:

Case 1: $\omega < \omega_c$: then complex term = 0 &

real term = $1 - 2\omega^2/\omega_c^2$

$\Rightarrow$ $A = 0$ and

$\cos B = 1 - 2\omega^2/\omega_c^2$

So wave propagates without attenuation.

Synthetic transmission-line

$$\left(\cosh(A)\cos(B) + j\sinh(A)\sin(B) = 1 - 2\omega^2/\omega_c^2 \right)$$

Case 2 $\omega > \omega_c$: then we can no longer have

$\cos B = 1 - 2\omega^2/\omega_c^2$ ← less than −1

hence $A \neq 0 \Rightarrow$ this means wave attenuates.

(note also $A \neq 0 \Rightarrow \sin B = 0 \Rightarrow B = \pi$)

So: Signals propagate only below the Bragg frequency $\omega_c = 2/\sqrt{LC}$

Below cutoff

Below cutoff: $A = 0$ $\quad \cos B = 1 - 2\omega^2/\omega_c^2$

$= 1 - \omega^2 LC/2$

Far below cutoff $B \ll 1$, $\cos B \cong 1 - B^2/2 + (B^4/24)$

↑ drop.

so: $1 - B^2/2 \cong 1 - \omega^2 LC/2$

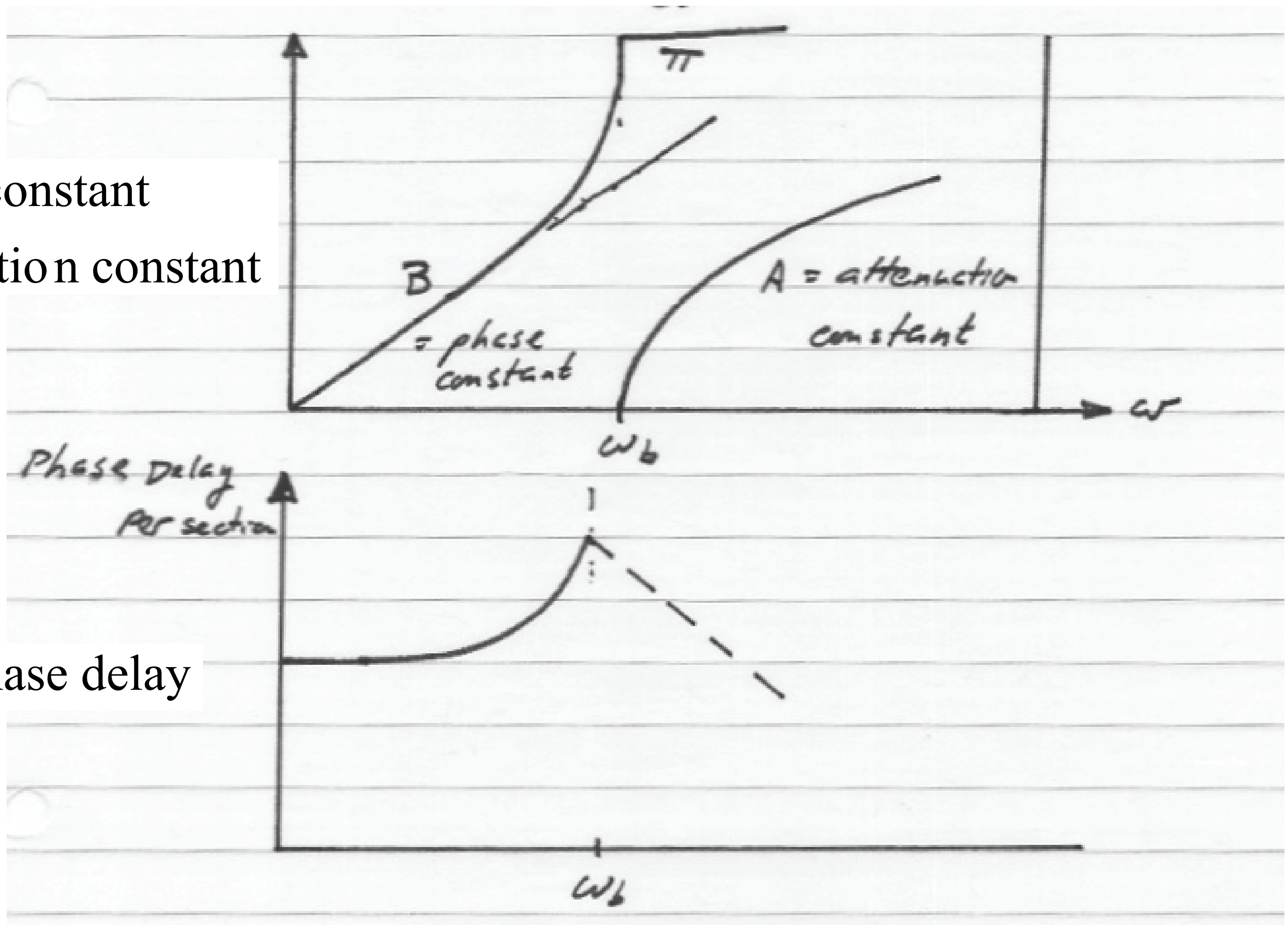
$$\Rightarrow B \cong \omega\sqrt{LC}$$

$$\text{phase delay} = \frac{B}{\omega} \cong \sqrt{LC}$$

Propagation characteristics

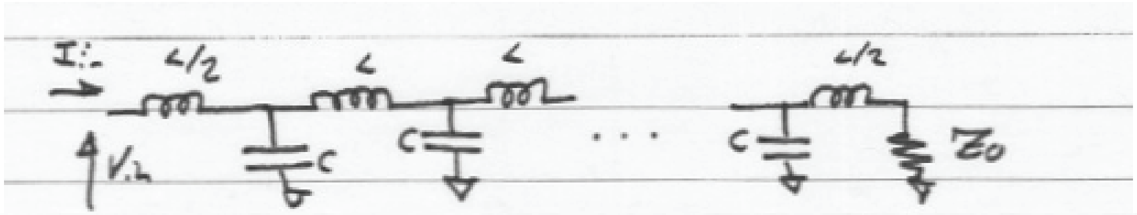
β : phase constant

α : attenuation constant

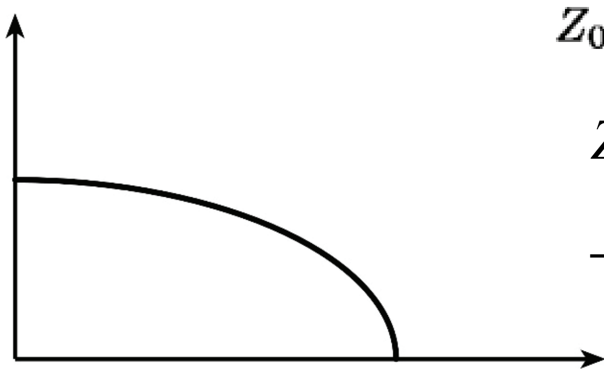


β / ω : phase delay

Characteristic Impedance



Defining $Z_0(\omega) = V_{in}(\omega)/I_{in}(\omega)$ a similar calculation can be made. (wont' repeat)

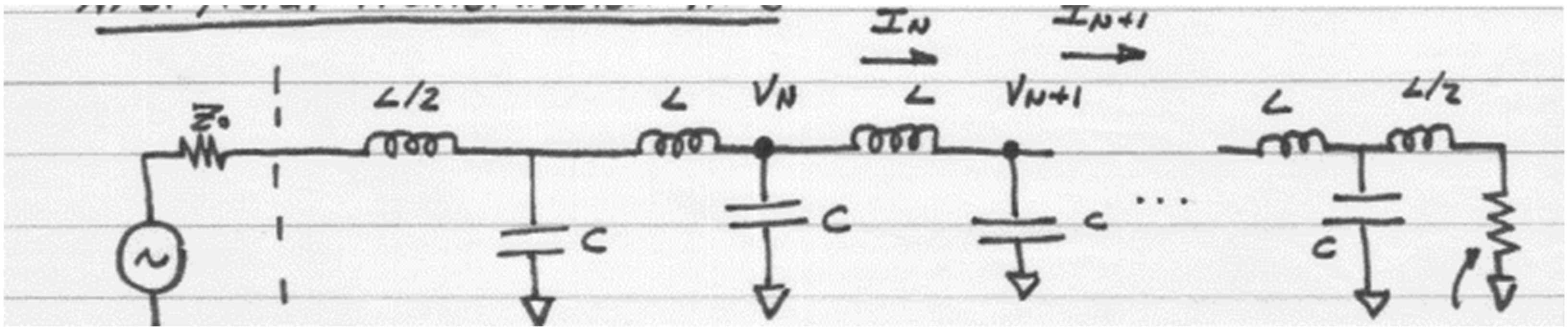


$$Z_0 \equiv V_{in}/I_{in} = \sqrt{Z_s/Y_p}\sqrt{1+Z_sY_p/4}.$$

$$Z_s = j\omega L, Y_p = j\omega C$$

$$\rightarrow Z_s = \sqrt{L/C}\cdot\sqrt{1-\omega^2/\omega_{Bragg}^2}$$

Synthetic transmission-line: summary

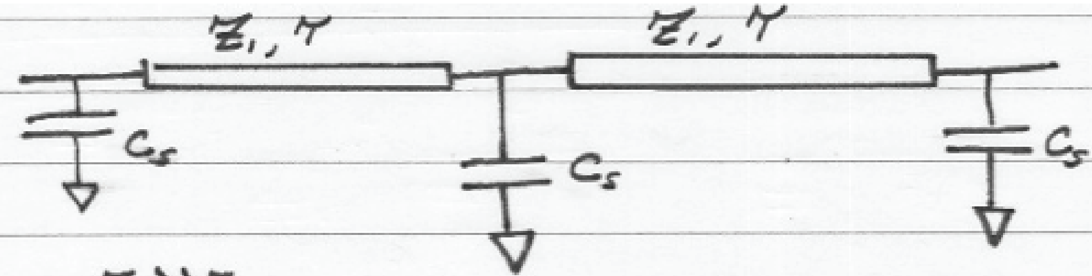


$Z_0 \cong \sqrt{L/C}$ for $\omega << \omega_c$

delay per section $\cong \sqrt{LC}$ for $\omega << \omega_c$

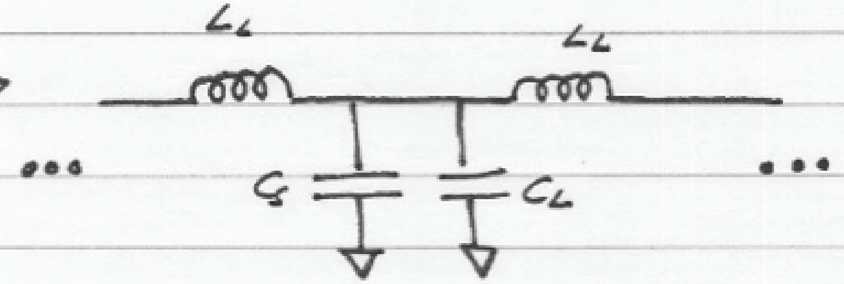
$\omega_c = \dfrac{2}{\sqrt{LC}}$ or $f_c = \dfrac{1}{\pi\sqrt{LC}}$

Realization with distributed elements



assume $Z_1 \gg Z_0$

$\cong \Rightarrow$



$L_L = \tau Z_1$

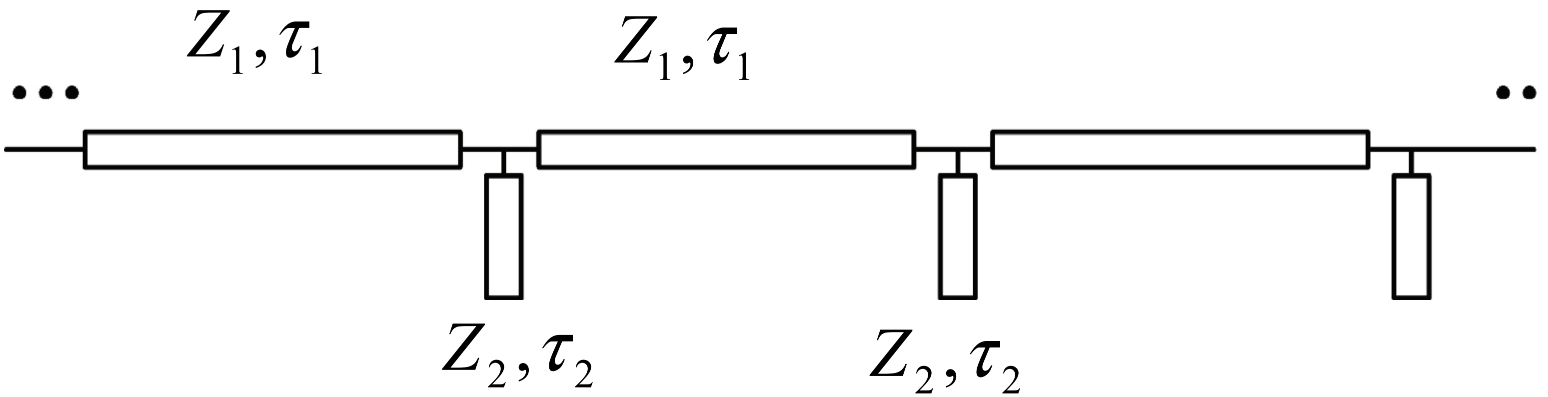
$C_L = \tau / Z_1$

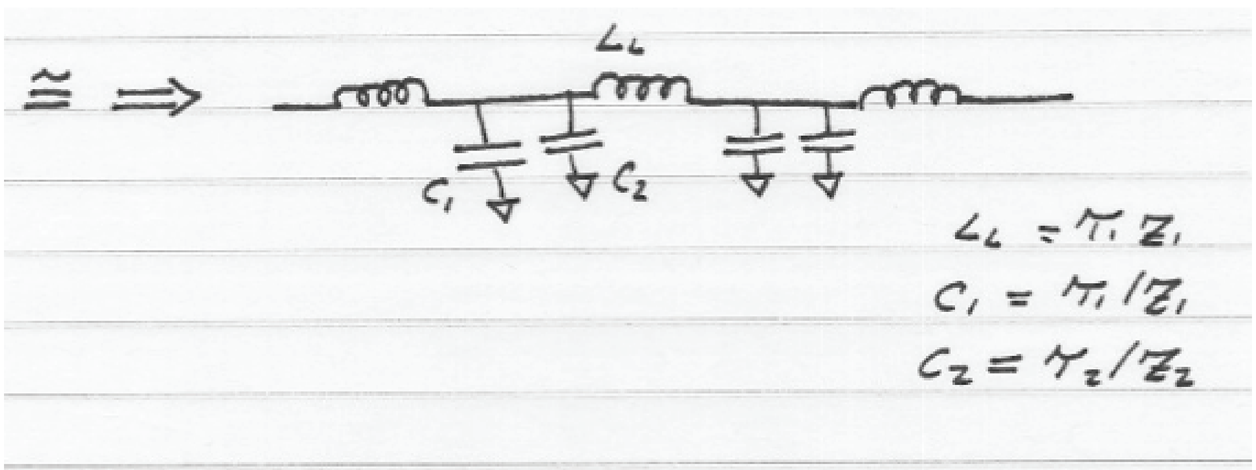
$$\Rightarrow \quad Z_0 \cong \sqrt{\frac{L_L}{C_s + C_L}} = \sqrt{\frac{\tau Z_1}{C_s + \tau / Z_1}} \quad \leftarrow \omega \ll \omega_c$$

$$T = \text{delay per section} \cong \sqrt{L_L(C_s + C_L)} = \sqrt{\tau Z_1 \left(C_s + \frac{\tau}{Z_1}\right)}$$

$$\omega_c \cong \frac{2}{\sqrt{L_L(C_s + C_L)}} = \frac{2}{\sqrt{\tau Z_1 (C_s + \tau / Z_1)}}$$

Another realization with distributed elements





These are rough approximations.

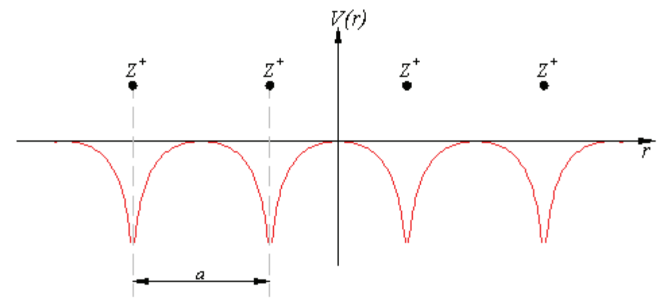
You can (1) do more careful derivations

or (2) check with a circuit simulator

Very broad applications of these concepts:

Kronig - Penney atomic lattice model

$\rightarrow$ band structure

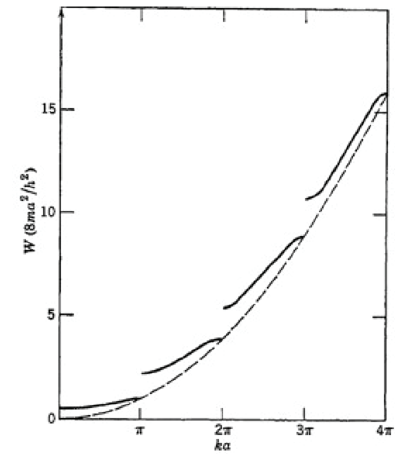


X - ray diffraction in crystals (Bragg)

https://en.wikipedia.org/wiki/Particle_in_a_one-dimensional_lattice

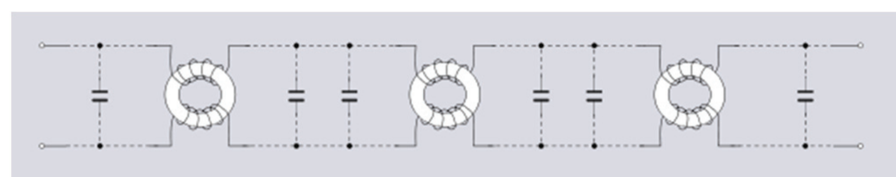
Wave propagation in periodic structures

$\rightarrow$ (1) filters and (2) physics



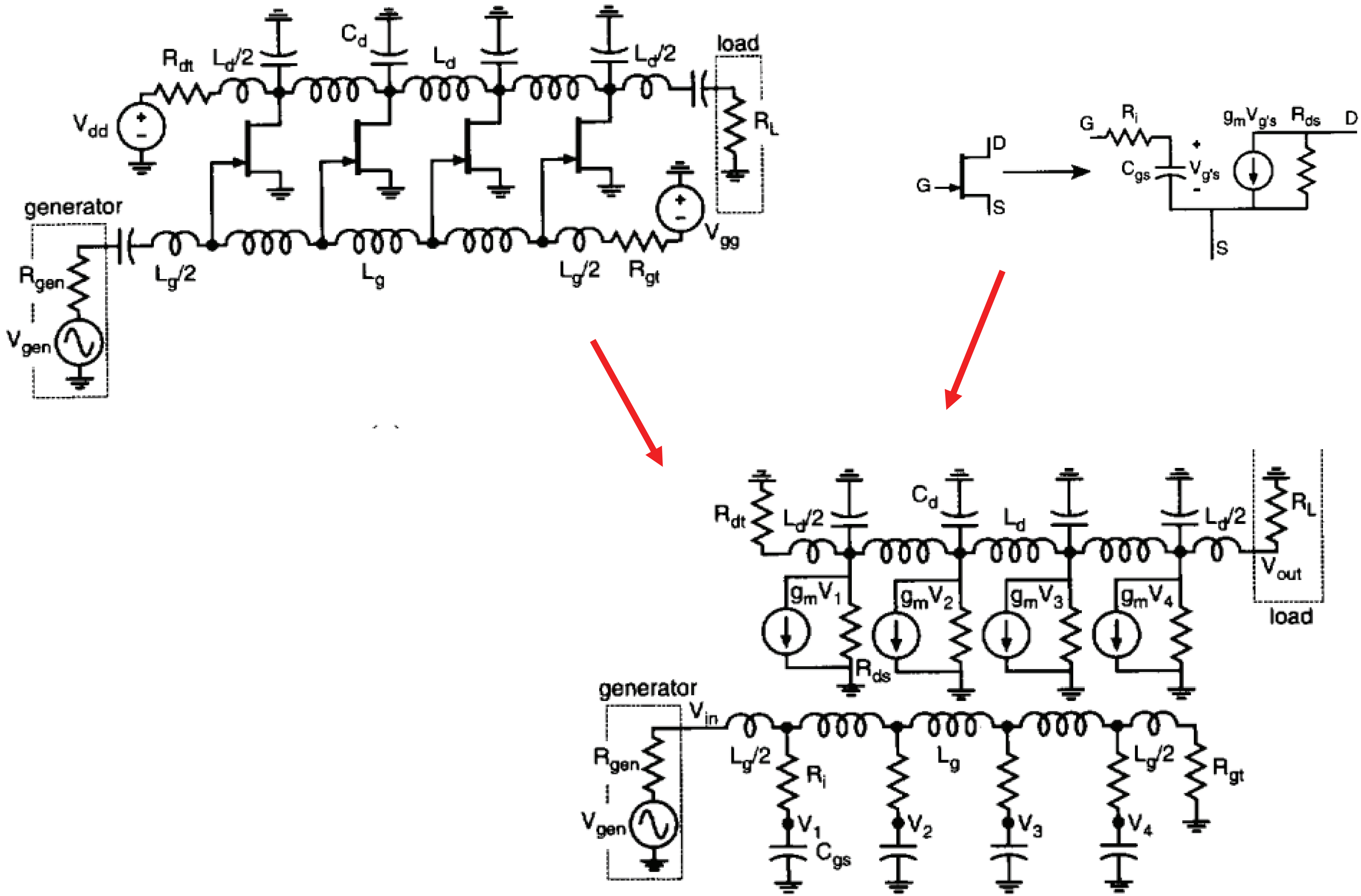
Earliest history : telegraph loading coils

Heaviside (1881)

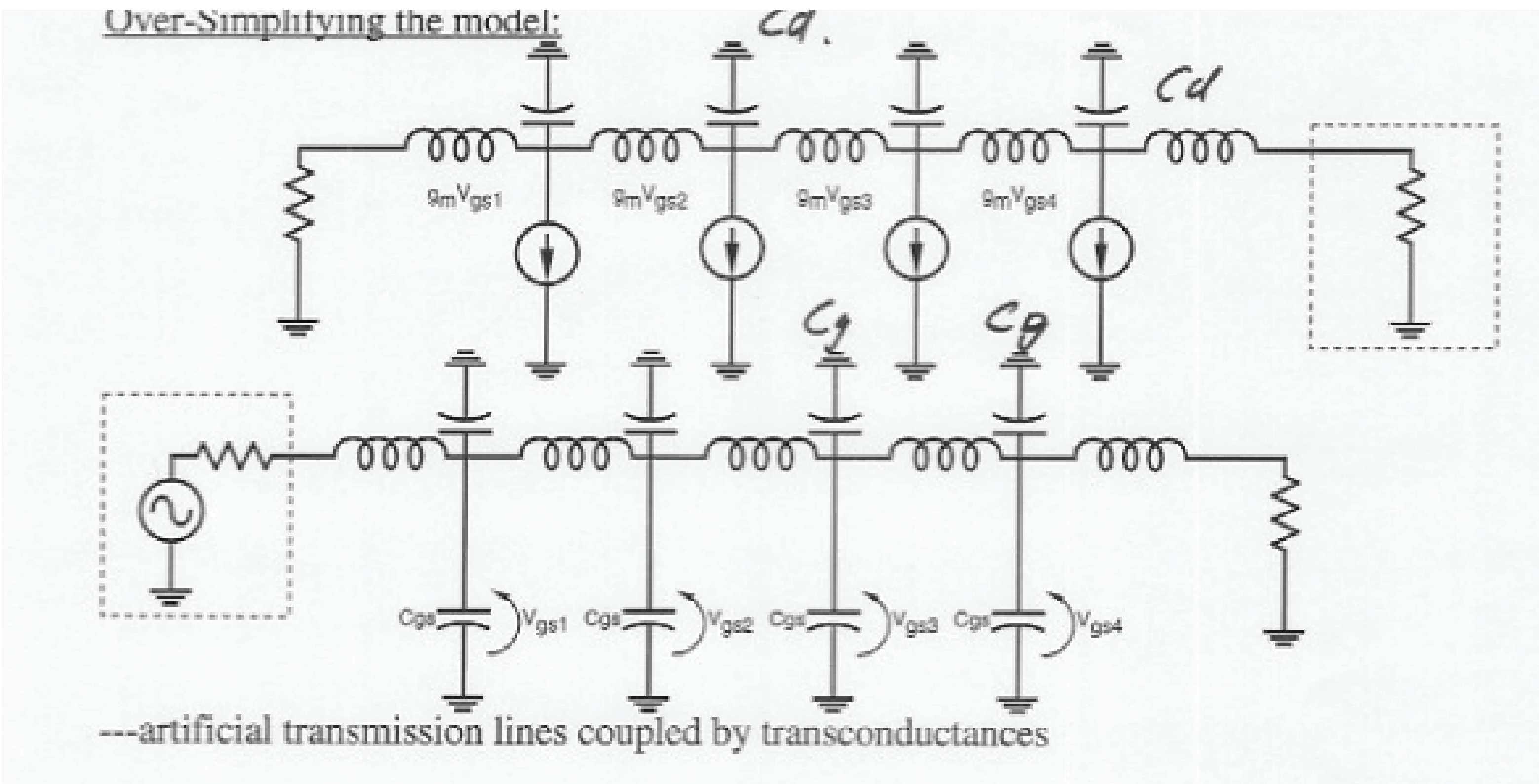


https://en.wikipedia.org/wiki/Loading_coil

Distributed Amplifier

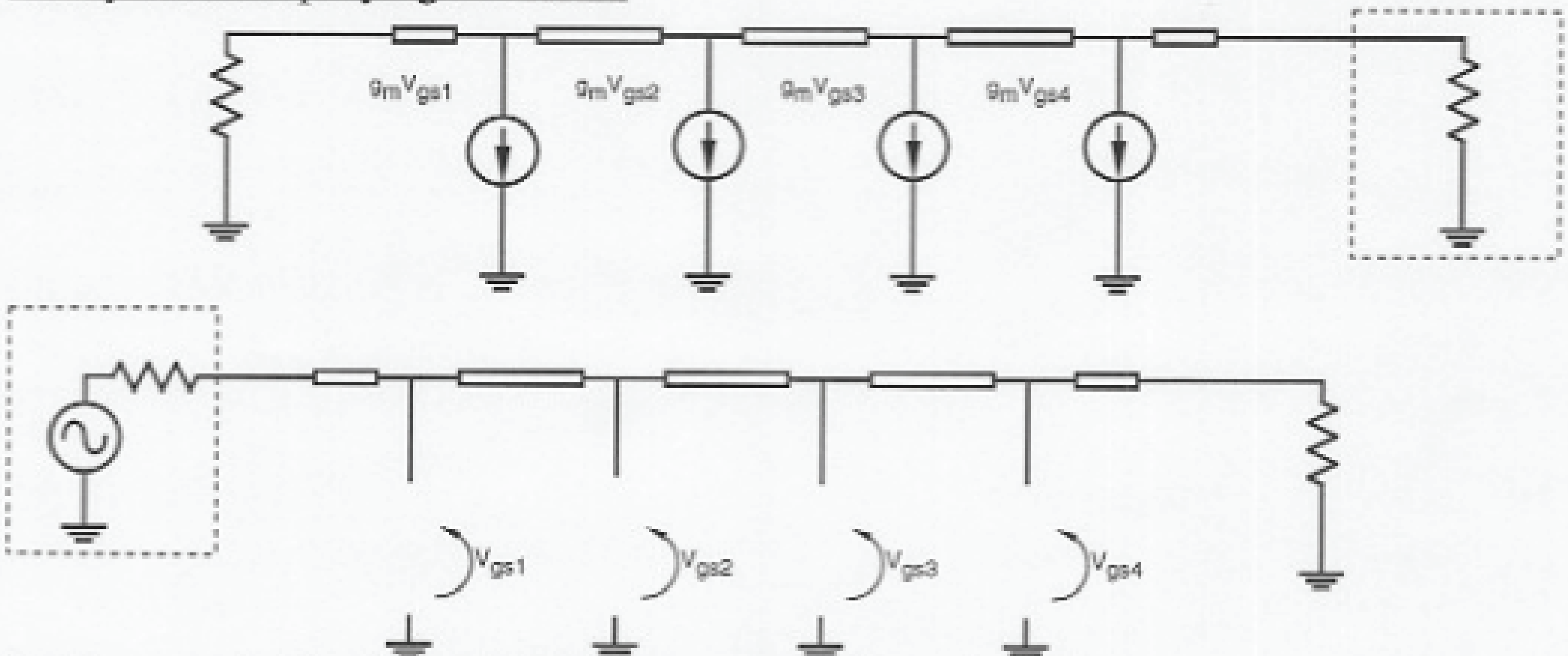


Over-simplifying the model:



Really over-simplifying the model:

Really-over simplifying the model

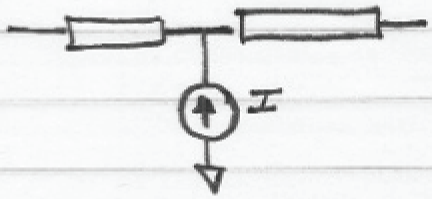


for frequencies $f \ll f_c$, artificial lines can be replaced by real ones,...

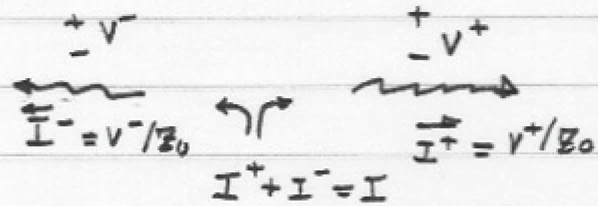
$T_d=\sqrt{(L_dC_d)}$ $\quad Z_d=\sqrt{(L_d/C_d)}$ $\quad T_g=\sqrt{(L_g(C_g+C_{gs}))}$ $\quad Z_g=\sqrt{(L_g/(C_g+C_{gs}))}$

Clearly we want Zd=Zg=50Ω; we also want Td=Tg.
Above model shows that bandwidth $\approx 1/\pi\sqrt{(L_g(C_g+C_{gs}))}$ if Tg=Td

Distributed amplifier gain



from symmetry, individual current generator develops equal waves in both directions.



Hence $I^+ = I/2$

and $V^+ = Z_0 I/2$

each sucessive device (FET) adds a current $I/2$ to the forward wave.

If they add in phase (If !!!)

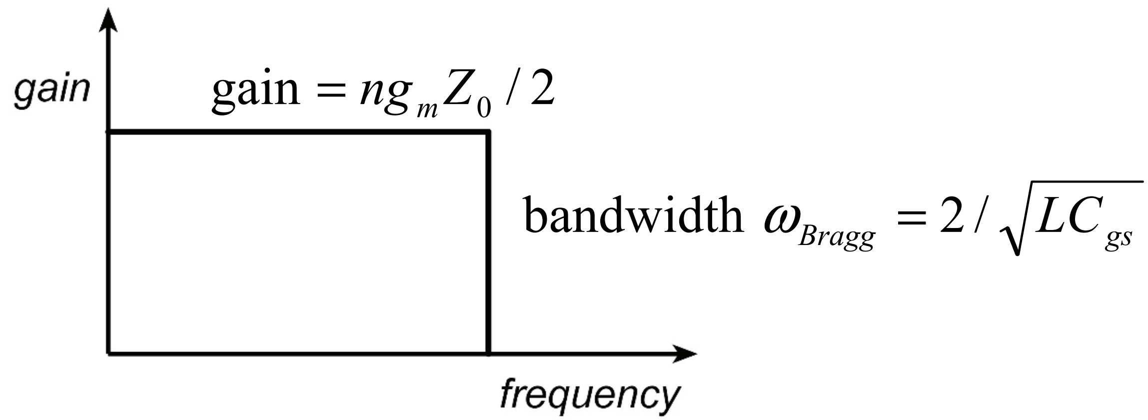
then total voltage after n devices

$$V^+ = n Z_0 I/2 = -\frac{n Z_0}{2} g_m V_{gs} = -\frac{n Z_0}{2} g_m V_{in}$$

Forward gain $= -n g_m Z_0/2$

Approaching the Bragg Frequency, the line will no longer transmit signals.

Distributed amplifier gain: simplest model



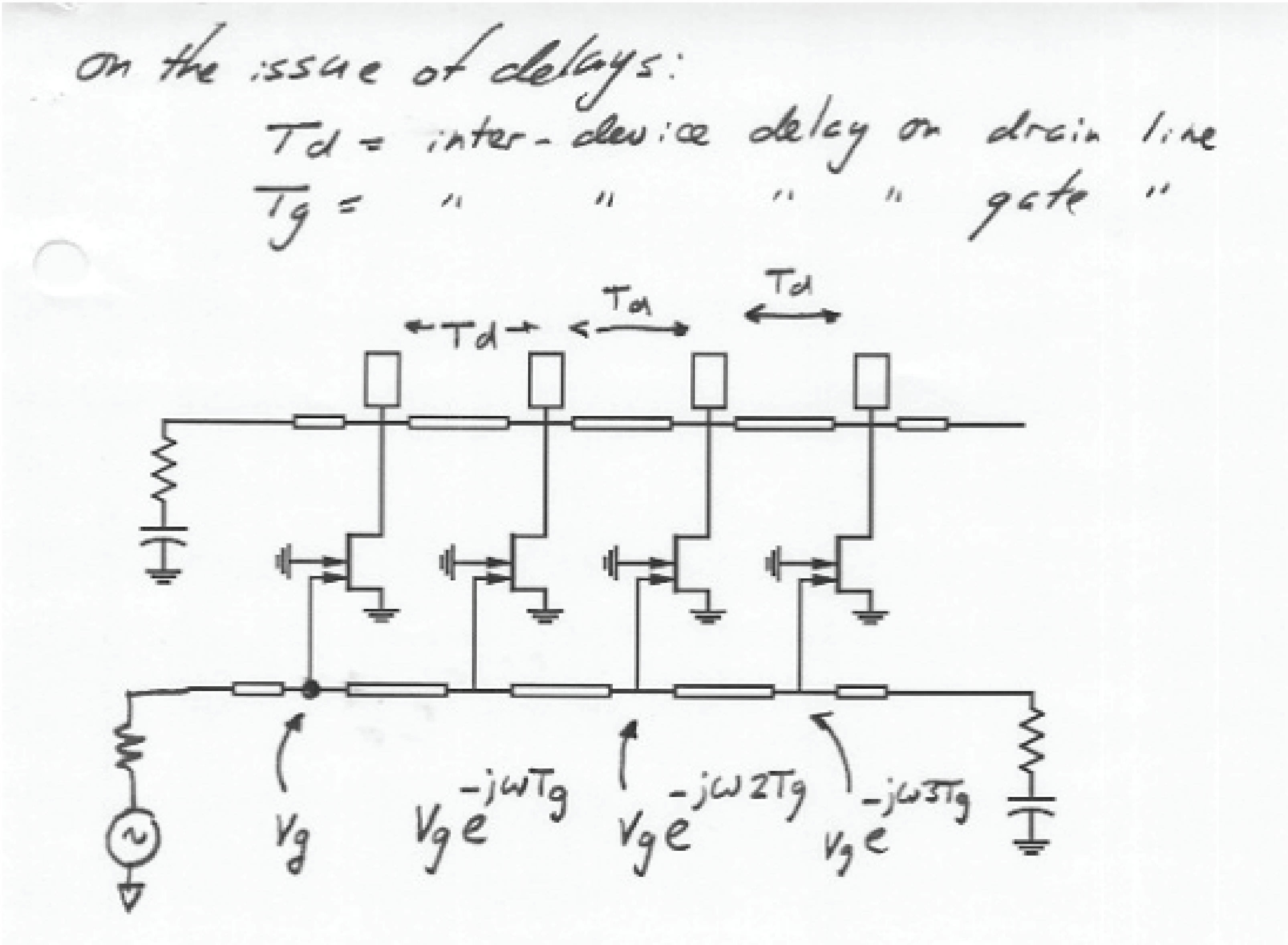
In this case, only the periodic lines limit the amplifier bandwidth.

Other limits :

velocity mismatch

losses (transistor and line resistances)

Delay mismatch



Delay mismatch

Neglect the delays of the first gate-line section and the last drain-line section, as these are common to all signals:

FET #1

gate 1 $\quad V_g e^{-j0\omega\tau_g}$

drain 1 $\quad g_m V_g e^{-j0\omega\tau_g}$

after reaching output: $-\frac{Z_0}{2} g_m V_g e^{-j0\omega\tau_g} e^{-j\omega 3\tau_d}$

FET #2

gate 1 $\quad V_g e^{-j\omega\tau_g}$

drain 1 $\quad g_m V_g e^{-j\omega\tau_g}$

after reaching output: $-\frac{Z_0}{2} g_m V_g e^{-j\omega\tau_g} e^{-j\omega 2\tau_d}$

Delay mismatch

Neglect the delays of the first gate-line section and the last drain-line section, as these are common to all signals:

FET #1

1 gate: $V_0 e^{-j0\omega\tau_g}$

Total output (4 Fets)

$$V_{out} = -V_g \cdot \frac{Z_0 g_m}{2}\left[e^{-j0\omega\tau_g} e^{-j3\omega\tau_d} + e^{-j\omega\tau_g} e^{-j\omega 2\tau_d} + e^{-j2\omega\tau_g} e^{-j\omega\tau_d} + e^{-j3\omega\tau_g} e^{-j\omega\tau_d} \right]$$

phase factors (or delays) must all be equal for outputs to add in-phase.

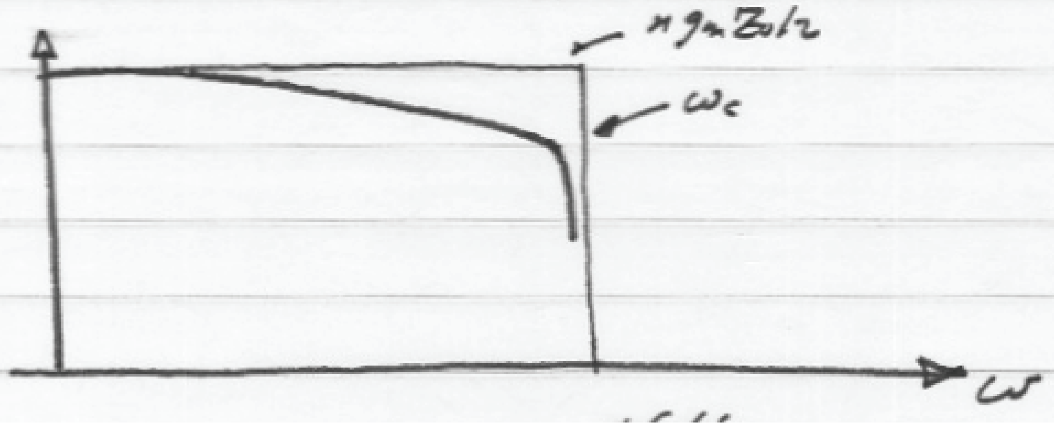
Delay mismatch

suppose T_g & T_d are mismatched by $\Delta T = T_d - T_g$

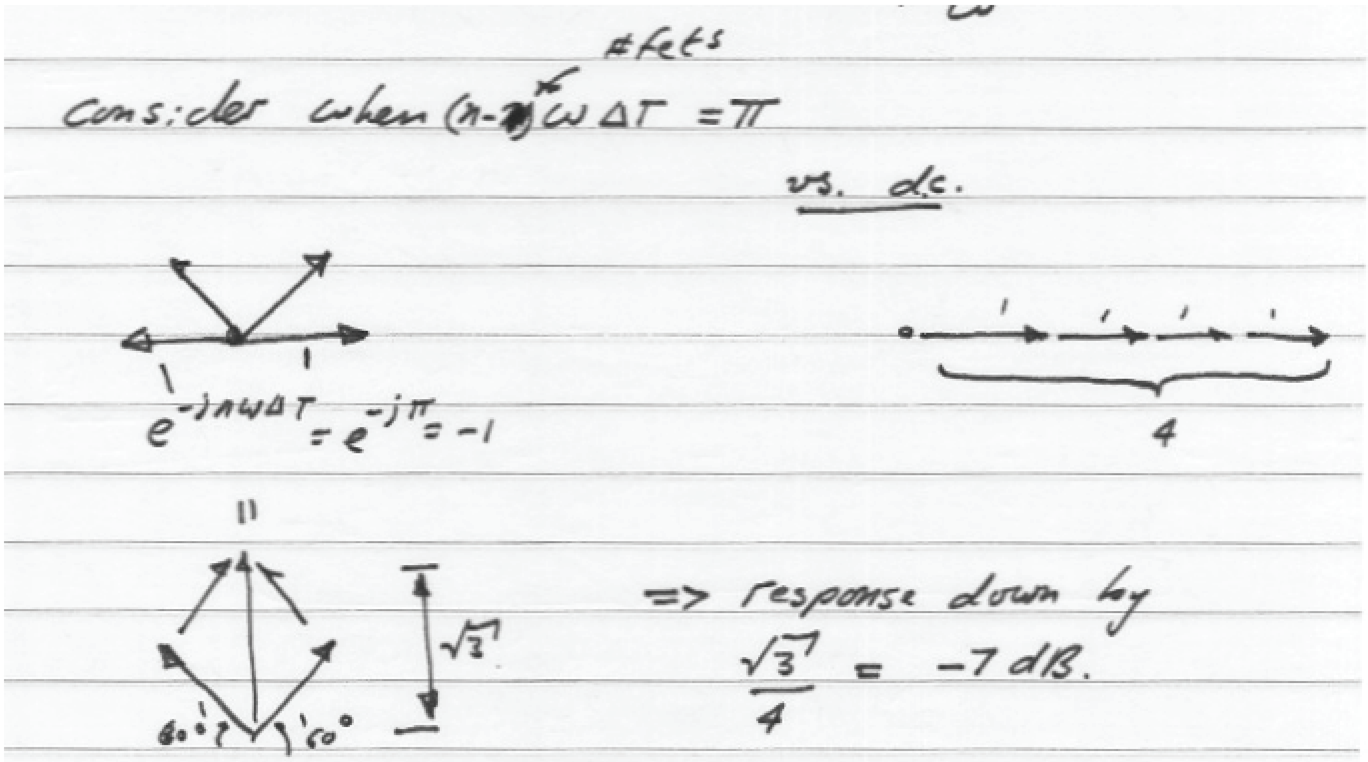
$$V_{out} = -V_g \frac{Z_0 g_m}{2} e^{-3j\omega T_g} \times$$ ← delay fixed, so who cares?

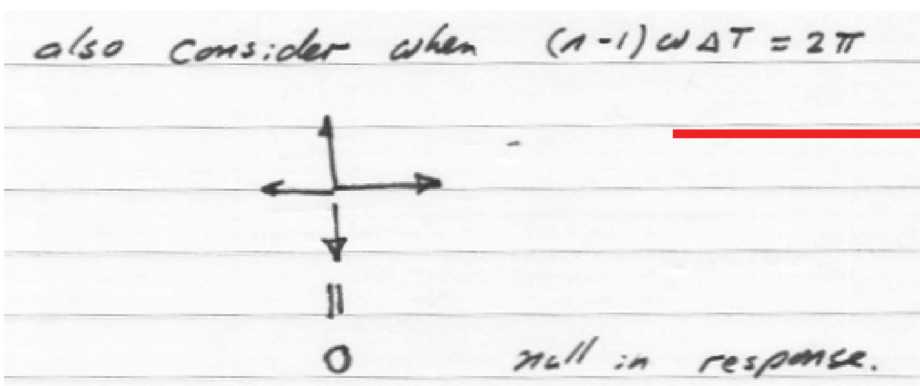
$$\left[e^{-j3\omega\Delta T} + e^{-j2\omega\Delta T} + e^{-j\omega\Delta T} + 1 \right]$$

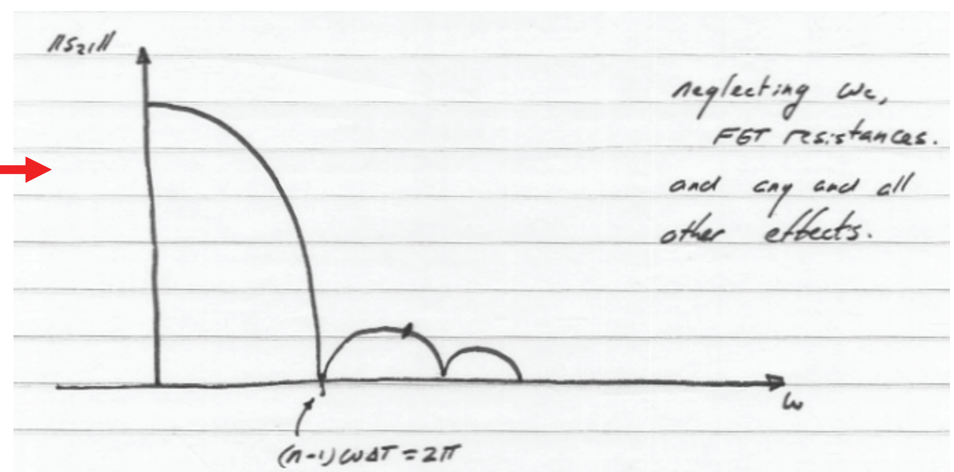
This will result in a rolloff in the frequency response:



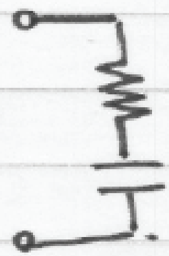
Delay mismatch→out-of-phase signals→rolloff







Input resistance: series-parallel transformation

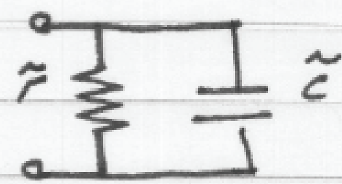
$Z \Rightarrow$ 

$$Z = r + 1/sC$$

$$Y = \frac{1}{Z} = \frac{1}{r + 1/sC} = \frac{sC}{1 + rsC} = \frac{j\omega c}{1 + j\omega rc}$$

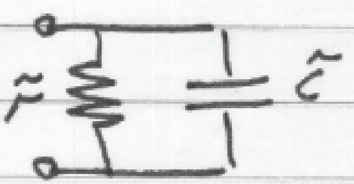
$$= \frac{j\omega c(1 - j\omega rc)}{(1 + \omega^2 r^2 c^2)} = \frac{j\omega c}{1 + \omega^2 r^2 c^2} + \frac{\omega^2 c^2 r}{1 + \omega^2 r^2 c^2}$$

$$= jb + g$$

$\Rightarrow$ 

$$\tilde{r} = \frac{1}{\omega^2 c^2 r}(1 + \omega^2 c^2 r^2)$$

$$\tilde{c} = c/(1 + \omega^2 c^2 r^2)$$

$\cong$ 

$$\left.\begin{aligned} \tilde{r} &\cong \frac{1}{\omega^2 c^2 r} \\ \tilde{c} &\cong c \end{aligned}\right\} \text{for } \omega \ll 1/rc \text{ (good approx)}$$

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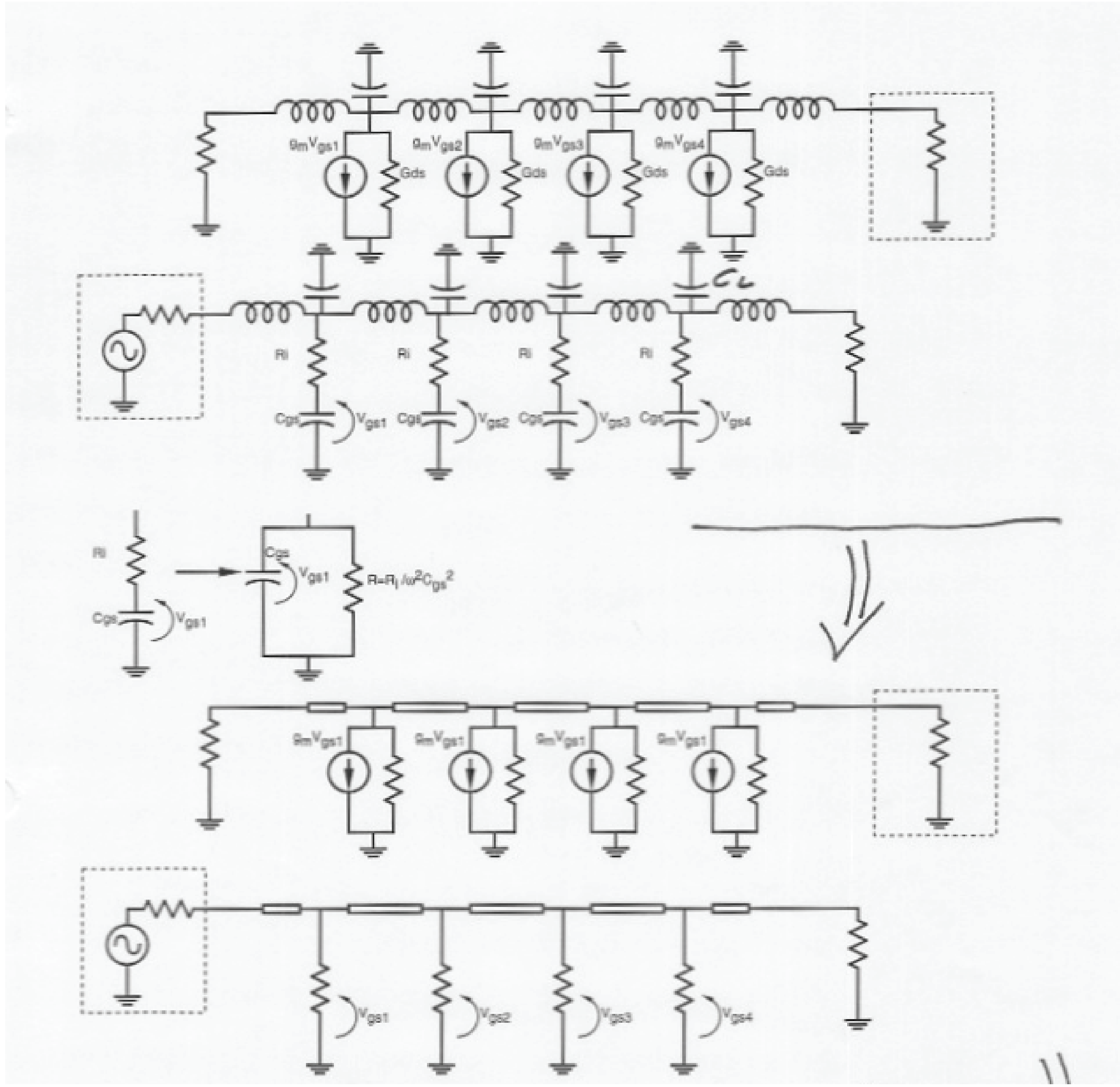
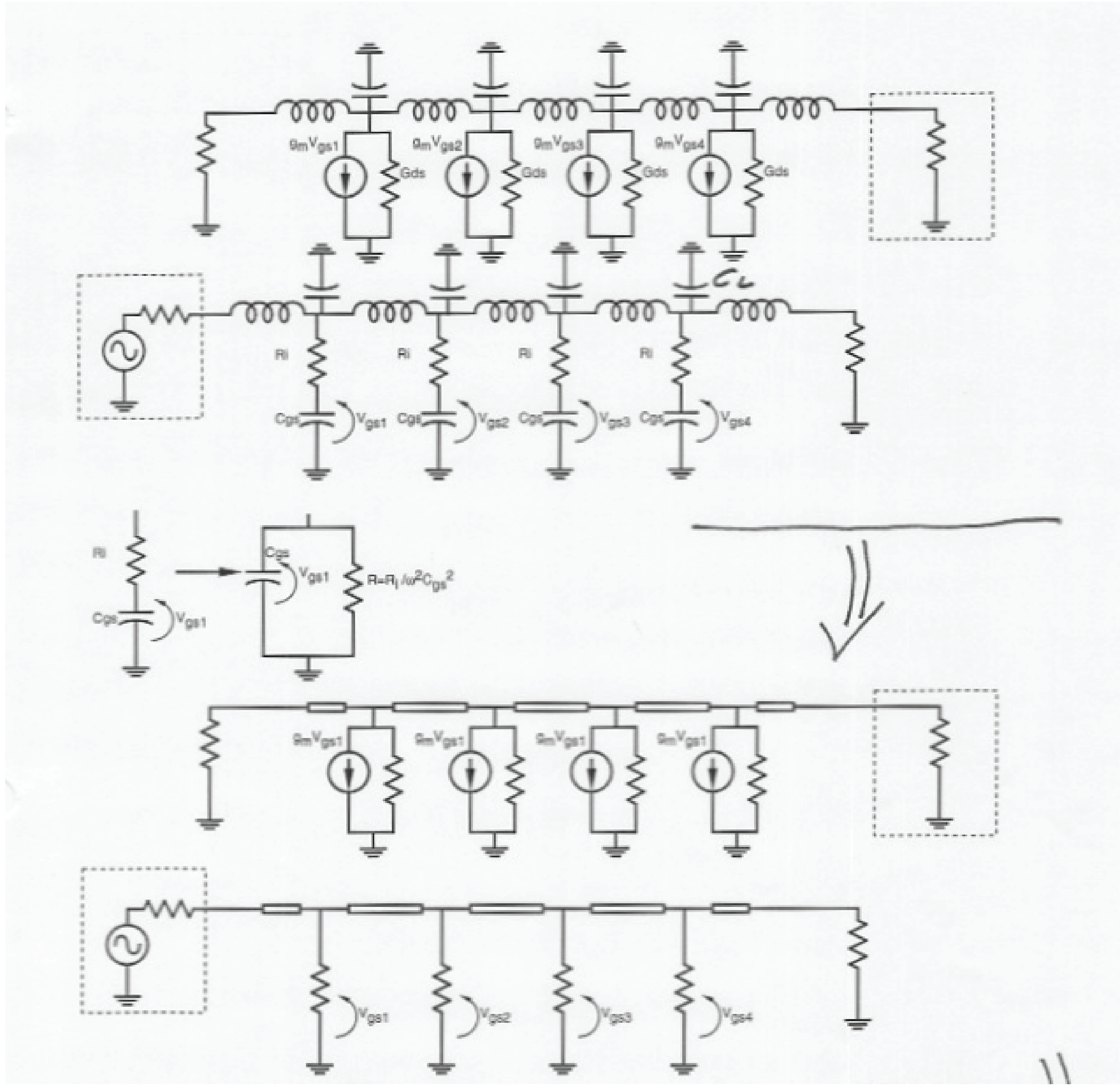
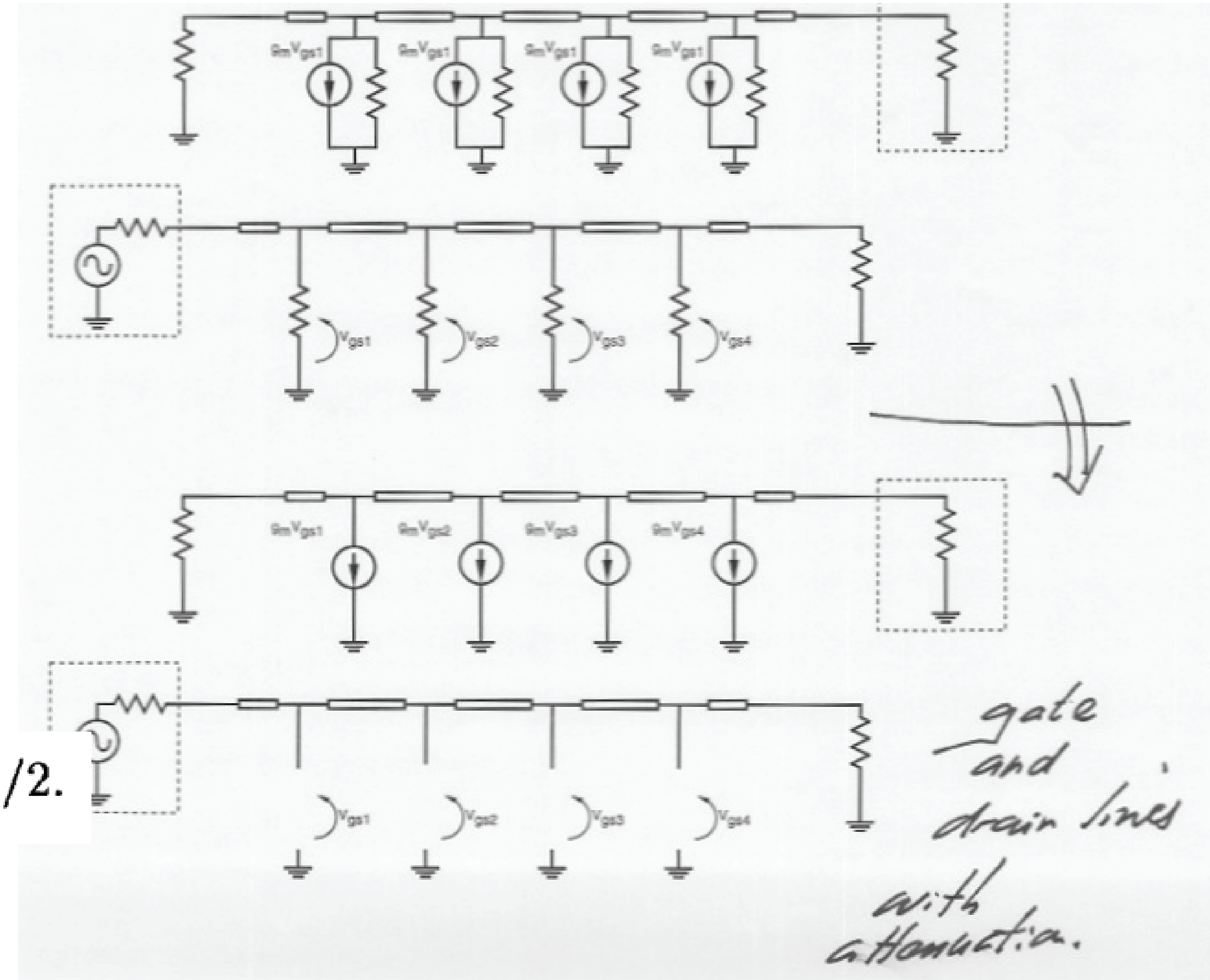
With transistor resistances (1):



With transistor resistances (1):



With transistor resistances (2):



$$\alpha_d \cong Z_d/2R_{\mathrm{ds}}.$$

$$\alpha_g \cong \omega^2 C_{\mathrm{gs}}^2 R_i Z_g/2.$$

Line losses and bandwidth

Attenuation per section on gate line:

$$A_g \cong \frac{Z_0}{2\tilde{r}} \quad \text{for } \tilde{r} >> Z_0$$

$$= \omega^2 C_{gs}^2 r_i Z_g / 2$$

Attenuation per section on drain line

$$A_d \cong \frac{Z_0}{2 r_{ds}} = Z_d G_{ds}/2 \quad \text{for } G_{ds} << 1/Z_0$$

Gate line attenuation generally dominates, particularly when dual-gate fets are used (very low Gds)

Line losses and bandwidth

Analyze, neglecting all other effects

Fet #1

gate 1: $V_g e^{-A_g/2}$ ← 1/2 line section

drain 1: $g_m V_g e^{-A_g/2}$

after reaching output:

$$-\frac{g_m Z_0}{2} V_g e^{-A_g/2} e^{-A_d(3\frac{1}{2})}$$

Fet #2

gate 2 $V_g e^{-A_g(1\frac{1}{2})}$

drain 2 $g_m V_g e^{-A_g 3/2}$

after reaching output:

$$-\frac{g_m Z_0}{2} V_g e^{-A_g \frac{3}{2}} e^{-A_d(2\frac{1}{2})}$$

etc

Total Output (4 fets)

$$V_{out} = -(V_g Z_0 g_m/2) \times e^{-A_g/2} e^{-A_d/2} \times$$

$$\left[e^{-3A_d} + e^{-A_g} e^{-2A_d} + e^{-2A_g} e^{-A_d} + e^{-3A_g} \right]$$

Line losses and bandwidth

A more simple interpretation :

Examining gate-line losses, the input voltage to the Nth transistor is attenuated by $e^{-(N-1/2)\alpha_g}$. Given a desired high-frequency cutoff ω_{high}, increasing the number of transistors beyond $N_{\max}$, given by

$$N_{\max}\omega_{\text{high}}^2 C_{\text{gs}}^2 R_i Z_g \simeq 1 \tag{5}$$

does not increase the high-frequency gain because transistors far from the input are not driven. This is Ayasli's criterion [42].

Examining the drain-line losses, the output of the 1st transistor is attenuated by $e^{-(N-1/2)\alpha_d}$. Increasing the number of transistors beyond $N_{\max}$ given by

$$N_{\max} Z_d / R_{ds} \simeq 1 \tag{6}$$

does not increase the amplifier gain because transistors near the input do not contribute to the amplifier output.

To summarize

- Analyze input & output structures as synthetic lines, determine $Z_g, Z_d, T_g, T_d, \omega_c$
- D.C. Gain is $-g_m n Z_0/2$
- Bandwidth limits are ω_c, delay mismatch on the 2 lines, and gate-line attenuation $\propto \omega^2$
- Design requirements of $Z_g = Z_d = Z_0$ and $T_g = T_d$ force $L_g = L_d$ and $C_d = C_{gs}$
- As $C_{gs} >> C_{output}$ for FETs, drain line must have the added capacitors to equalize delays

within the bandwidth, we have:

$$\omega < \omega_c$$

$$n\omega^2 C_{gs}^2 r_i Z_g \leq 1$$

$$(n-1)\omega \Delta T \leq \pi/2$$

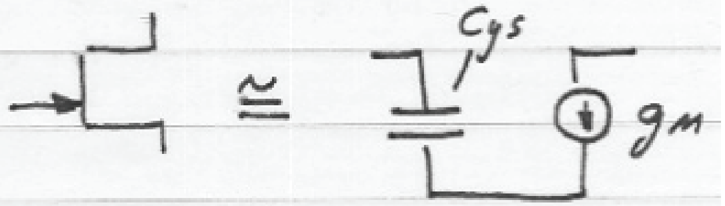
$$A_0 \cong -n g_m Z_0/2$$

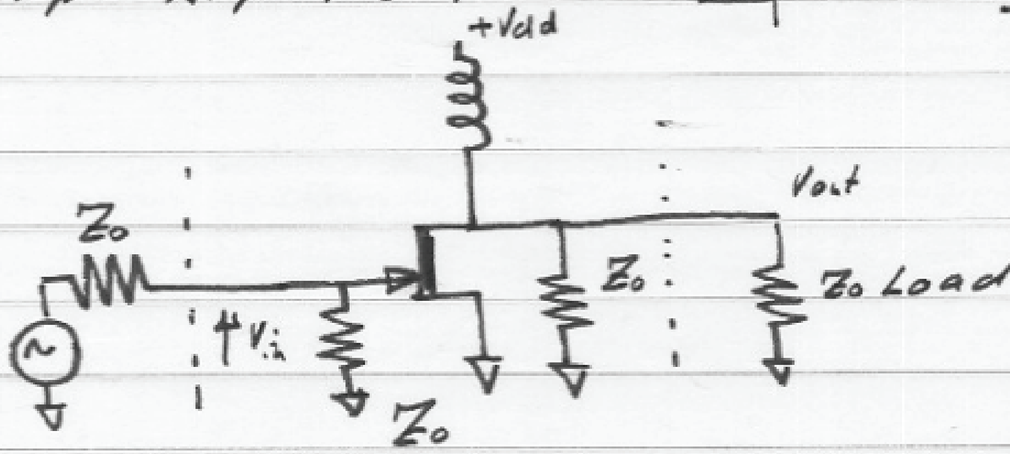
also $n Z_d G_{ds} \leq 1$

Why use distributed amplifiers ?

Answer: Increased gain-bandwidth product without the use of matching (resonant) networks.

Simple Amplifier:





A_0 = DC gain = $g_m Z_0/2$

ω_m = bandwidth = $\dfrac{1}{C_{gs} Z_0/2}$

gain-bandwidth product

= $A_0 \omega_m$ = $g_m / C_{gs} 2\pi$ = f_τ

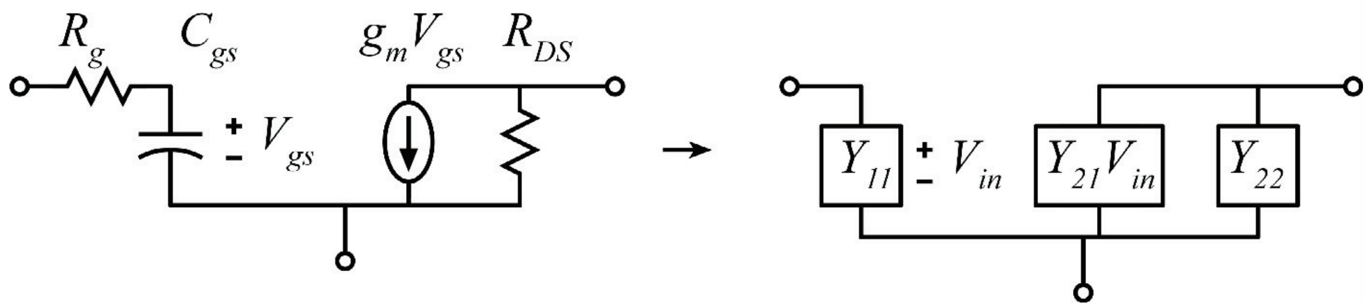
Increase transistor size : g_m, C_{gs} will increase. Inceased gain, decreased bandwidth

gain - bandwidth produce remains at f_τ (less when all parasitics are included)

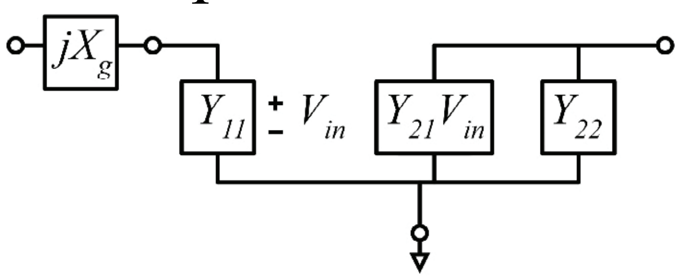
Bandwidth of distributed amplifier

First : consider two forms of degeneration.

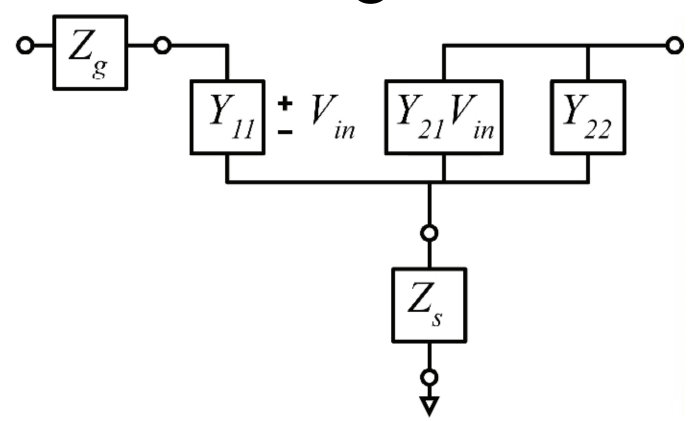
Model of unilateral device



input attenuation



source degeneration

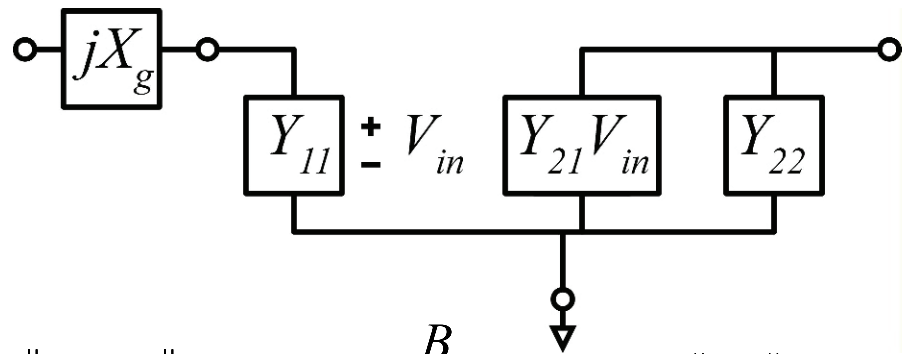


For brevity : analyze only input attenuation

Source degenration with parallel RC combination also useful

Input degernation

Overall network Y - parameters : $Y_{new,ij}$



$$Y_{new,21} = \frac{1/Y_{11}}{1/jB_g + 1/Y_{11}} \cdot Y_{21} = \frac{jB_g}{G_{11} + jB_g + jB_{11}} \cdot Y_{21} \rightarrow \|Y_{new,21}\| = \frac{B_g}{\left(G_{11}^2 + (B_g + B_{11})^2\right)^{1/2}} \cdot \|Y_{21}\|$$

$$Y_{new,11} = \frac{1}{jX_g + 1/Y_{11}} = \frac{jB_g Y_{11}}{jB_g + Y_{11}} = \frac{jB_g(G_{11} + jB_{11})}{G_{11} + jB_g + jB_{11}} = \frac{jB_g(G_{11} + jB_{11})(G_{11} - jB_g - jB_{11})}{\left(G_{11}^2 + (B_g + B_{11})^2\right)}$$

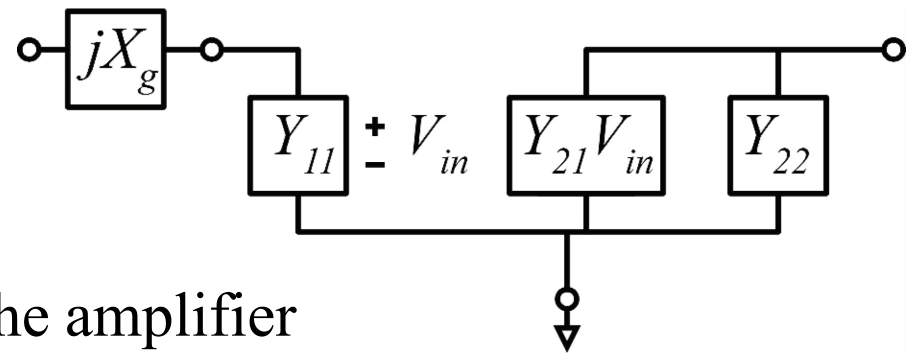
$$Y_{new,11} = \frac{jB_g(G_{11}^2 - jG_{11}B_g - B_{11}(B_{11} + B_g))}{\left(G_{11}^2 + (B_g + B_{11})^2\right)}$$

$$G_{new,11} = \frac{B_g^2 G_{11}}{\left(G_{11}^2 + (B_g + B_{11})^2\right)} = G_{11} \cdot \frac{\|Y_{new,21}\|^2}{\|Y_{21}\|^2} \quad \text{and} \quad G_{new,22} = G_{22}$$

G_{11} is reduced, G_{22} is unchanged, $\|Y_{21}\|$ is reduced,

and, as expected (jX_g is lossless), $G_{max} = \|Y_{21}\|^2 / 4G_{11}G_{22}$ is unchanged

Input degernation with distributed amplifier



Suppose we apply degeneration to make

$$G_{11,new} = G_{22,new}$$

at the upper 3-dB design frequency of the amplifier

Line losses per section :

$$\alpha_{\text{in}} = G_{new,11} Z_0 / 2,\ \alpha_{\text{out}} = G_{22} Z_0 / 2 = G_{new,11} Z_0 / 2$$

Maximum # sections : $n_{\max}\alpha = 1/2$

$$\rightarrow n_{\max} G_{new,11} Z_0 = n_{\max} G_{22} Z_0 = 1$$

Circuit power gain

$$\|S_{21}\|^2 = \frac{n_{\max}^2 \|Y_{new,21}\|^2 Z_0^2}{4} = \frac{\|Y_{new,21}\|^2}{4 G_{new,11} G_{22}} = \frac{\|Y_{21}\|^2}{4 G_{11} G_{22}} = G_{\max}$$

Maximum feasible TWA gain-bandwidth

Circuit power gain

$$\left\|S_{21}\right\|^2 = \frac{n_{\max}^2 \left\|Y_{new,21}\right\|^2 Z_0^2}{4} = \frac{\left\|Y_{new,21}\right\|^2}{4G_{new,11}G_{22}} = \frac{\left\|Y_{21}\right\|^2}{4G_{11}G_{22}} = G_{\max}$$

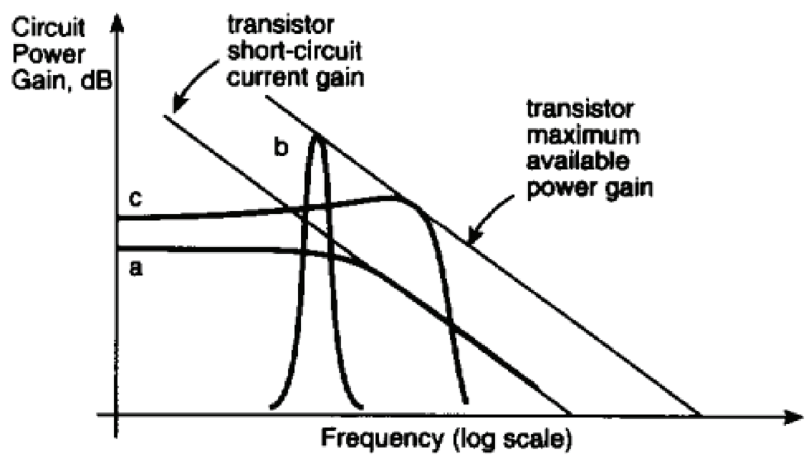


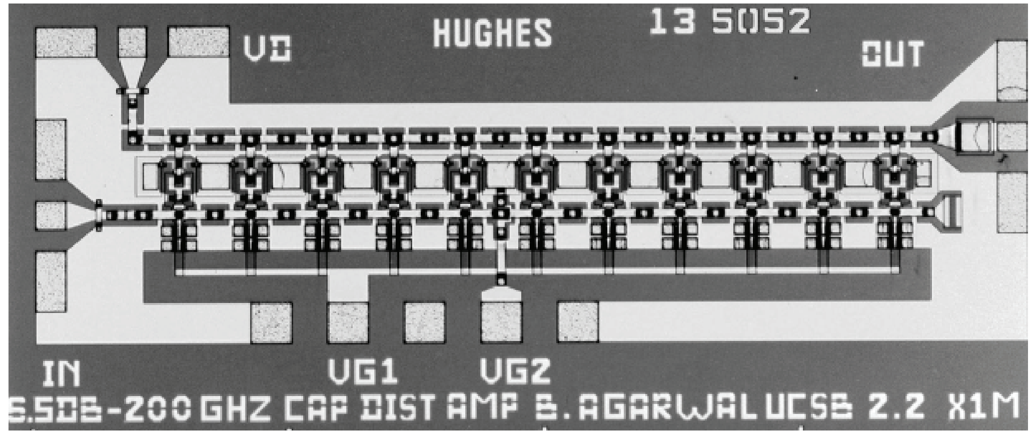
Fig. 8. Gain–frequency constraints of wideband lumped-element (*a*), resonant (*b*), and distributed (*c*) amplifiers .

A perfectly - designed TWA can attain gain = $G_{\max}$ at the amplifier upper bandwidth limit

Examples

112-GHz, 157-GHz, and 180-GHz InP HEMT Traveling-Wave Amplifiers

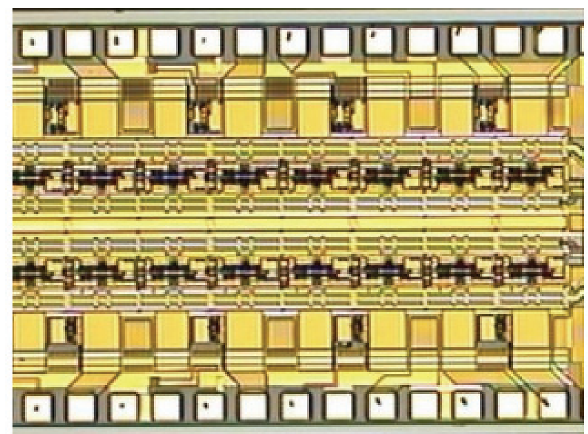
Agarawal, Trans. MTT, 12/1998



0.1–42 GHz InP DHBT distributed amplifiers with 35 dB gain and 15 dBm output

Modulator driver for 40Gb/s optical links

Krishnamurthy, Electronics letters, 2003

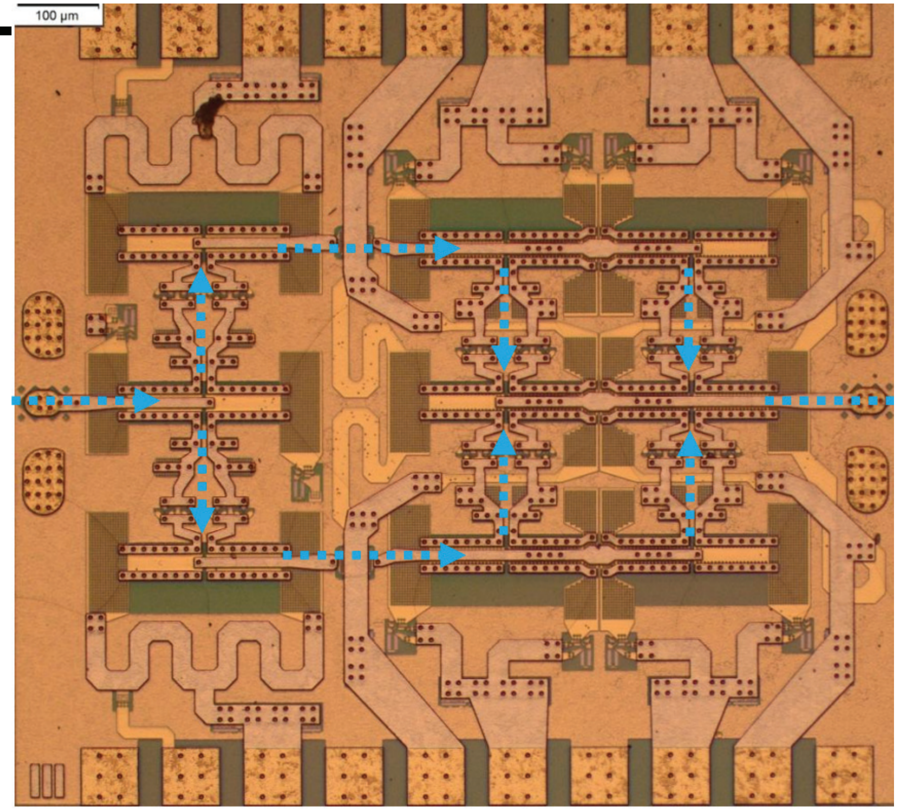


Examples

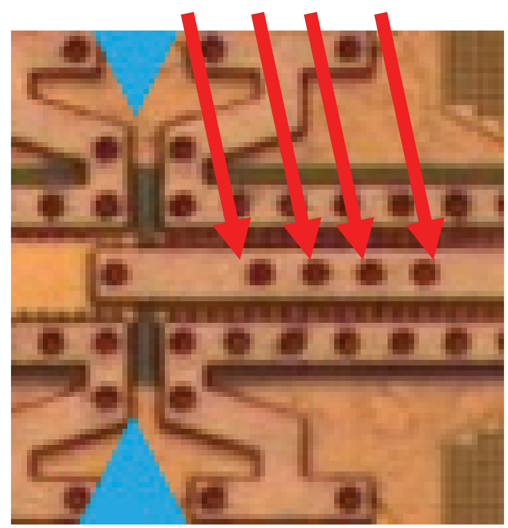
In this power amplifier,
we needed lines with $Z_0 = 25\Omega$

A25Ω line woudl have
been much too wide

$\rightarrow$ realized using
synthetic line



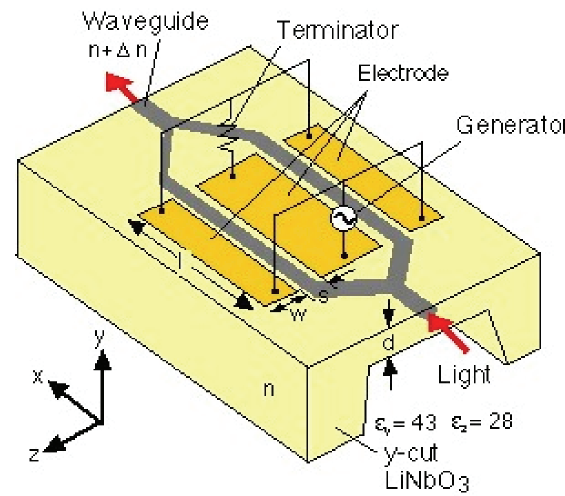
arrows point to vias connnectin g
to MIM capacitors



86GHz power amplifier
Park, Trans. MTT, 10/2014

Examples

traveling wave optical modulator



traveling wave tube