ECE 145C / 218C, notes set xx: Receiver Sensitivity

Mark Rodwell
Doluca Family chair
University of California, Santa Barbara

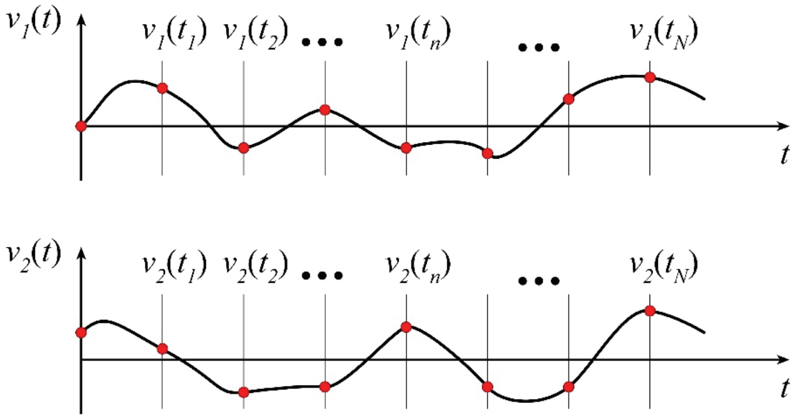
rodwell@ece.ucsb.edu

Waveforms as Vectors

Consider signals $v_1(t)$ and $v_2(t)$ bandwidth-limited to DC-f_{high}

Sample these at Nyquist intervals $t_1, t_2 = t_1 + \Delta t, t_3 = t_2 + \Delta t$ ... where $\Delta t = 1/2 f_{\text{high}}$

$v_1(t)$ becomes the vector $\vec{v}_1 = \begin{bmatrix} v_1(t_1) \\ v_1(t_2) \\ \vdots \\ v_1(t_N) \end{bmatrix}$ while $v_2(t)$ becomes the vector $\vec{v}_2 = \begin{bmatrix} v_2(t_1) \\ v_2(t_2) \\ \vdots \\ v_2(t_N) \end{bmatrix}$



Dot product or vector projection: $\langle \vec{v}_1 | \vec{v}_2 \rangle = v_1(t_1)v_2(t_1) + v_1(t_2)v_2(t_2) + \cdots + v_1(t_N)v_2(t_N)$

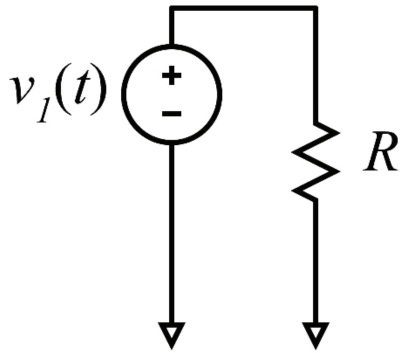
Limit as $\Delta t \to 0$ (or as $f_{high} \to \infty$):

$\langle v_1(t) | v_2(t) \rangle = \int_{-T/2}^{T/2} v_1(t)v_2(t)dt$ dot product between two signals

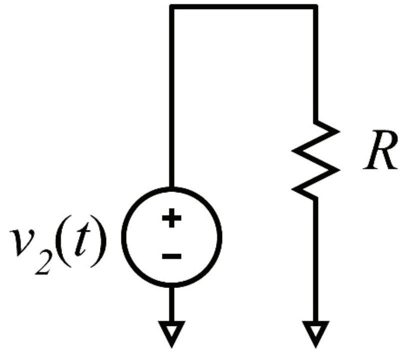
The time between $-T/2$ and $+T/2$ is the duration of our experiment

Signal energies, orthogonal signals

Two signals $v_1(t)$ and $v_2(t)$ are orthogonal (perpendicular) if $\langle v_1(t) | v_2(t) \rangle = \int_{-T/2}^{T/2} v_1(t) v_2(t) dt = 0$



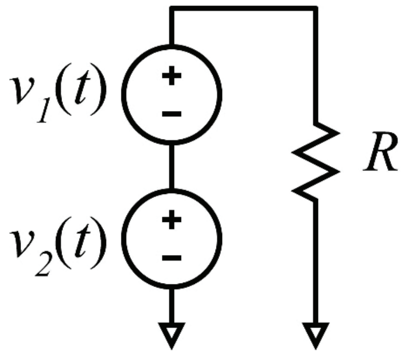
Energy due to $v_1(t)$: $E_1 = \frac{1}{R} \cdot \int_{-T/2}^{T/2} v_1^2(t) dt \rightarrow E_1 = \frac{1}{R} \cdot \langle v_1(t) | v_1(t) \rangle$



Energy due to $v_2(t)$: $P_2 = \frac{1}{R} \cdot \int_{-T/2}^{T/2} v_2^2(t) dt \rightarrow E_2 = \frac{1}{R} \cdot \langle v_2(t) | v_2(t) \rangle$

Energy due to $v_1(t)$ and $v_2(t)$:

$$E_{12} = \frac{1}{R} \cdot \int_{-T/2}^{T/2} \left(v_1(t) + v_2(t) \right)^2 dt = \frac{1}{R} \cdot \int_{-T/2}^{T/2} \left(v_1^2(t) + v_2^2(t) + 2 v_1(t) v_2(t) \right) dt = E_1 + E_2 + \frac{2}{R} \langle v_1(t) | v_2(t) \rangle$$



$E_{12} = E_1 + E_2$ if and only if $\langle v_1(t) | v_2(t) \rangle = 0$

Radio transmitter

In each symbol period $0,\ t_s,\ 2t_s,...$

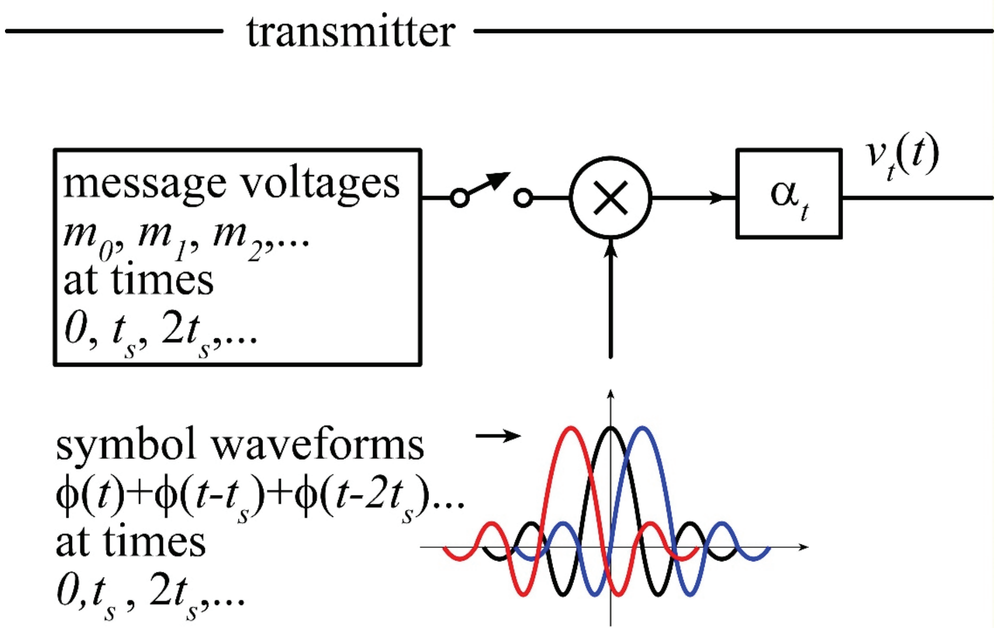
we apply message voltages $m_0, m_1, m_2,...$

Each of these are multiplied in the transmitter by the symbol waveforms,

$$\phi_{total}(t) = \phi(t) + \phi(t-t_s) + \phi(t-2t_s) + ...$$

The transmitted voltage is then amplified by α_t :

$$v_t(t) = m_1\alpha_t\phi(t) + m_2\alpha_t\phi(t-t_s) + m_3\alpha_t\phi(t-2t_s) + ...$$



Simplify: use orthogonal waveforms between symbol periods:

$$\langle\phi(t)|\phi(t-t_s)\rangle = \langle\phi(t)|\phi(t-2t_s)\rangle = ... = 0$$

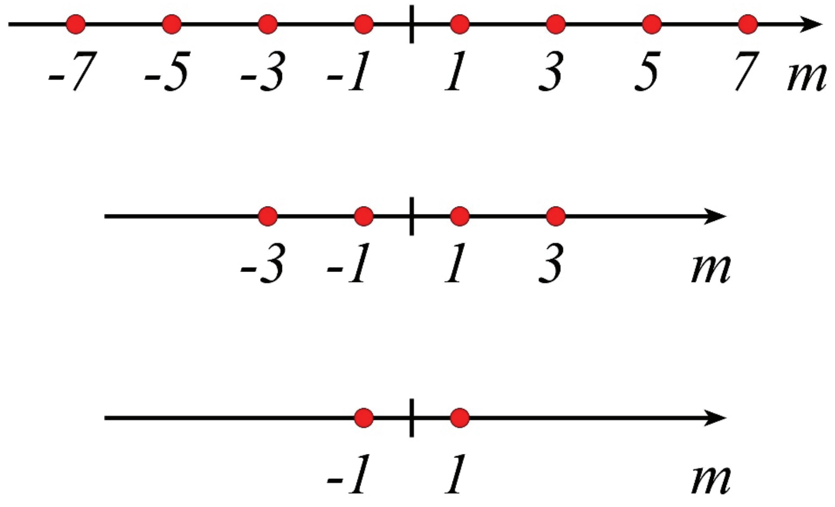
Flat* channel frequency response $\rightarrow$ zero intersymbol interference $\rightarrow$ can analyze one symbol period at a time.

*If the channel response is not flat, we lose orthogonality between symbol periods. $\rightarrow$ Intersymbol interference. Need equalization. Much more complex to analyze. Much more complex to design.

Transmitted Signal Energy for One Symbol Period

Normalization: symbol waveform energy $= E_\phi = R^{-1}\cdot\langle \phi(t) | \phi(t)\rangle = 1$ Joule

Transmit waveform voltage for one symbol period: $v_{t,n}(t) = \alpha_t m_n \phi(t - nt_s)$



Transmit signal energy for one symbol period

$$E_t = R^{-1}\cdot\langle v_{t,n}(t) | v_{t,n}(t)\rangle = \alpha_t^2 \cdot \|m^2\|\ E_\phi = \alpha_t^2\cdot\|m^2\|\cdot(1\text{ Joule})$$

m might be -1/+1 (1 bit/symbol) $\rightarrow \|m^2\| = 1$

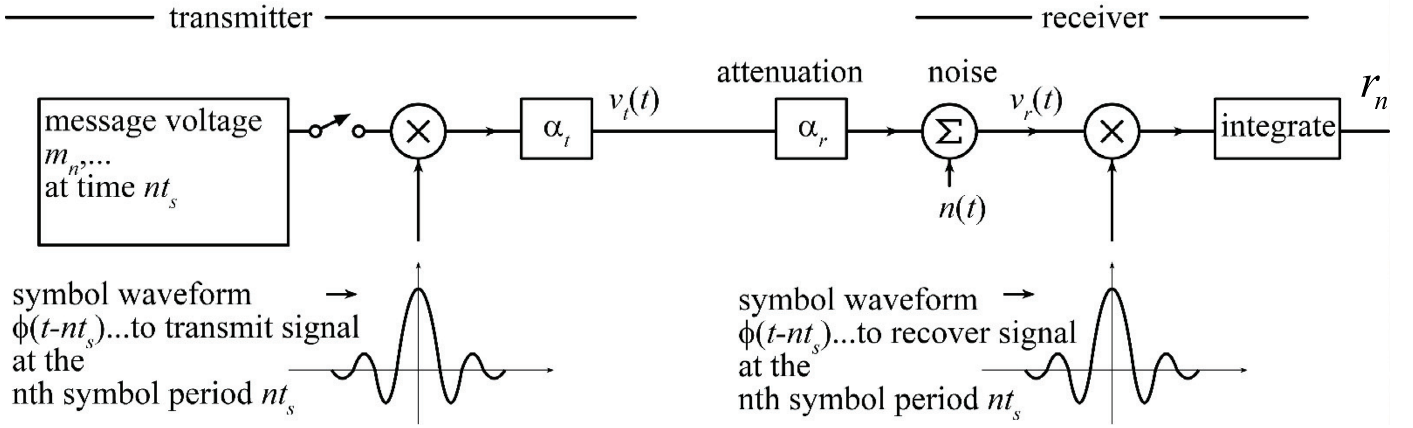
m might be $-3/-1/+1/+3$ (2 bits/symbol) $\rightarrow \|m^2\| = 0.25(9+1+1+9) = 5$

m might be -7/$-5/-3/-1/+1/+3/+5/7$ (3 bits/symbol) $\rightarrow \|m^2\| = (1/8)(49+25+9+1+1+9+25+49) = 168/8 = 21$

Radio channel model: orthogonal symbol waveforms

The channel attenuates the signal by $\alpha_r : 1$ in voltage

The receiver adds noise $n(t) \rightarrow \; v_r(t) = \alpha_r v_t(t) + n(t)$



To recover the signal in the n^{th} time slot,

the receiver correlates $v_r(t)$ with $\phi(t-nt_s)$,

then divides by R, and divides by $E_\phi^{1/2}$

$$r_n = \frac{1}{RE_\phi^{1/2}} \langle v_r(t) | \phi(t-nt_s) \rangle = \frac{1}{RE_\phi^{1/2}} \int_{-\infty}^{\infty} v_r(t)\phi(t-nt_s)dt$$

We divide by $RE_\phi^{1/2}$ because this causes r_n to have units of (Joules)$^{1/2}$.

which will provide key insights for sensitivity analysis

$$r_n = R^{-1}E_\phi^{-1/2} \langle v_r(t) | \phi(t-nt_s) \rangle = R^{-1}E_\phi^{-1/2} \langle \alpha_r \alpha_t m_n \phi(t-nt_s) + n(t) | \phi(t-nt_s) \rangle$$

$$r_n = \alpha_r \alpha_t m_n R^{-1}E_\phi^{-1/2} \langle \phi(t-nt_s) | \phi(t-nt_s) \rangle + R^{-1}E_\phi^{-1/2} \langle n(t) | \phi(t-nt_s) \rangle$$

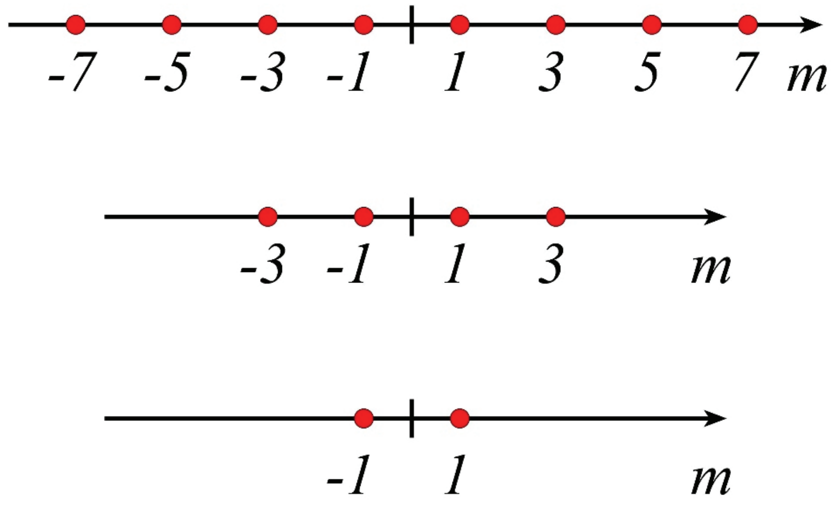
$$r_n = \alpha_r \alpha_t m_n E_\phi^{+1/2} + R^{-1}E_\phi^{-1/2} \langle n(t) | \phi(t-nt_s) \rangle = \alpha m_n E_\phi^{+1/2} + R^{-1}E_\phi^{-1/2} \langle n(t) | \phi(t-nt_s) \rangle$$

Where we have written $\alpha = \alpha_r \alpha_t$ to de-clutter the math

*If the channel response is not flat, we lose orthogonality between symbol periods. → Intersymbol interference. Need equalization. Much more complex.

Received Signal Energy for One Symbol Period

Normalization: symbol waveform energy = $E_\phi = R^{-1} \cdot \langle \phi(t) | \phi(t) \rangle = 1$ Joule



Received waveform voltage for one symbol period: $v_{r,signal,n}(t) = \alpha m_n \phi(t - nt_s)$

Received signal energy for one symbol period

$$E_r = R^{-1} \cdot \langle \alpha m_n \phi(t - nt_s) | \alpha m_n \phi(t - nt_s) \rangle = \alpha^2 \cdot \| m^2 \| \; E_\phi = \alpha^2 \cdot \| m^2 \| \cdot (1 \text{ Joule})$$

m might be -1/+1 (1 bit/symbol) $\rightarrow \| m^2 \| = 1$

m might be $-3/-1/+1/+3$ (2 bits/symbol) $\rightarrow \| m^2 \| = 0.25(9+1+1+9) = 5$

m might be -7/$-5/-3/-1/+1/+3/+5/7$ (3 bits/symbol) $\rightarrow \| m^2 \| = (1/8)(49+25+9+1+1+9+25+49) = 168/8 = 21$

Receiver noise 1

Received signal for one symbol period

$$r_n = \alpha m_n E_\phi^{+1/2} + R^{-1} E_\phi^{-1/2} \langle n(t) | \phi(t - nt_s) \rangle = r_{signal,n} + r_{noise,n}$$

Noise in n^{th} timeslot; $r_{noise,n} = R^{-1} E_\phi^{-1/2} \langle n(t) | \phi(t - nt_s) \rangle$ Noise in m^{th} timeslot; $r_{noise,m} = R^{-1} E_\phi^{-1/2} \langle n(t) | \phi(t - mt_s) \rangle$

Correlation of noise between symbol periods

$$E\left[r_{noise,n} r_{noise,m} \right] = E_\phi^{-1} R^{-2} \cdot E\left[\langle n(t) | \phi(t - nt_s) \rangle \langle n(t) | \phi(t - mt_s) \rangle \right]$$

$$= E_\phi^{-1} R^{-2} \cdot E\left[\int n(t)\phi(t - nt_s)dt \cdot \int n(\tau)\phi(\tau - mt_s)d\tau \right]$$

$$= E_\phi^{-1} R^{-2} \cdot E\left[\iint n(t)n(\tau)\phi(t - nt_s)\phi(\tau - mt_s)dtd\tau \right]$$

$$= E_\phi^{-1} R^{-2} \cdot \iint E[n(t)n(\tau)]\phi(t - nt_s)\phi(\tau - mt_s)dtd\tau$$

$$= E_\phi^{-1} R^{-2} \cdot \iint R_{nn}(t - \tau)\phi(t - nt_s)\phi(\tau - mt_s)dtd\tau$$

where $R_{nn}(t - \tau)$ is the autocorrelation of $n(t)$

But $n(t)$ has a power spectral density of $S_{nn}(j2\pi f) = kTFR/2$ (V^2 / Hz, **double-sided**)

where F is the system noise figure.

So, $R_{nn}(\tau) = (kTFR/2) \cdot \delta(\tau)$,

because $R_{nn}(\tau)$ and $S_{nn}(j2\pi f)$ are a Fourier transform pair.

Receiver noise 2

$$E\left[r_{noise,n}r_{noise,m}\right]=E_\phi^{-1}R^{-2}\cdot\iint R_{nn}(t-\tau)\phi(t-nt_s)\phi(\tau-mt_s)\,dt\,d\tau$$

$$= E_\phi^{-1}R^{-2}\cdot\iint (kTF/2)R\cdot\delta(t-\tau)\phi(t-nt_s)\phi(\tau-mt_s)\,dt\,d\tau.$$

$$=E_\phi^{-1}R^{-2}\cdot\int (kTF/2)R\cdot\phi(t-nt_s)\phi(t-mt_s)\,dt$$

$$=E_\phi^{-1}(kTF/2)R^{-1}\cdot\int \phi(t-nt_s)\phi(t-mt_s)\,dt=E_\phi^{-1}kTFR^{-1}\left\langle\phi(t-nt_s)\middle|\phi(t-mt_s)\right\rangle$$

but $R^{-1}\left\langle\phi(t-nt_s)\middle|\phi(t-mt_s)\right\rangle=\begin{cases}E_\phi=1\text{ Joule} & \text{for } n=m\\ 0\text{ Joule} & \text{for } n\neq m\end{cases}$

So $E\left[r_{noise,n}r_{noise,m}\right]=\begin{cases}kTF/2 & \text{for } n=m\\ 0 & \text{for } n\neq m\end{cases}$

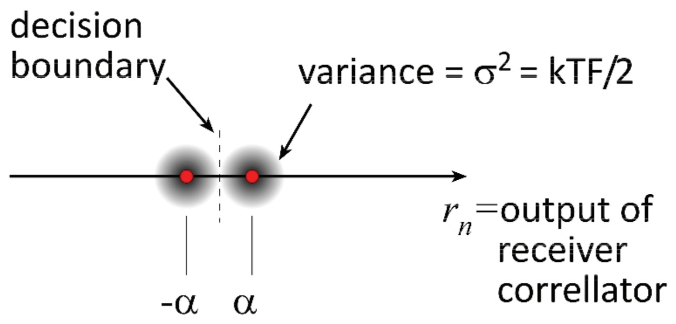
(we had chosen our original scaling of r and E_ϕ to get this magnitude)

Bit error rate vs. SNR: binary modulation

Received signal plus noise for nth symbol period: $r_n = r_{signal,n} + r_{noise,n}$

$$\sigma^2_{noise,n} = E\left[r_{noise,n} r_{noise,n}\right] = kTF/2 \rightarrow \sigma_{noise,n} = \sqrt{kTF/2}$$

$$r_{signal,n} = \alpha m_n E_\phi^{+1/2} = \alpha m_n \cdot (1 \text{ Joule})$$



Case 1, binary signals: $m_n = -1, +1$, $\|m^2\| = 1$

$m_n = -1$:	$m_n = +1$:
Gaussian with mean= $-\alpha$,	Gaussian with mean= α,
standard deviation= $\sigma = \sqrt{kTF/2}$	standard deviation= $\sigma = \sqrt{kTF/2}$

We assume that $m_n = 1$ if $r_n > 0$ and that $m_n = -1$ if $r_n < 0$

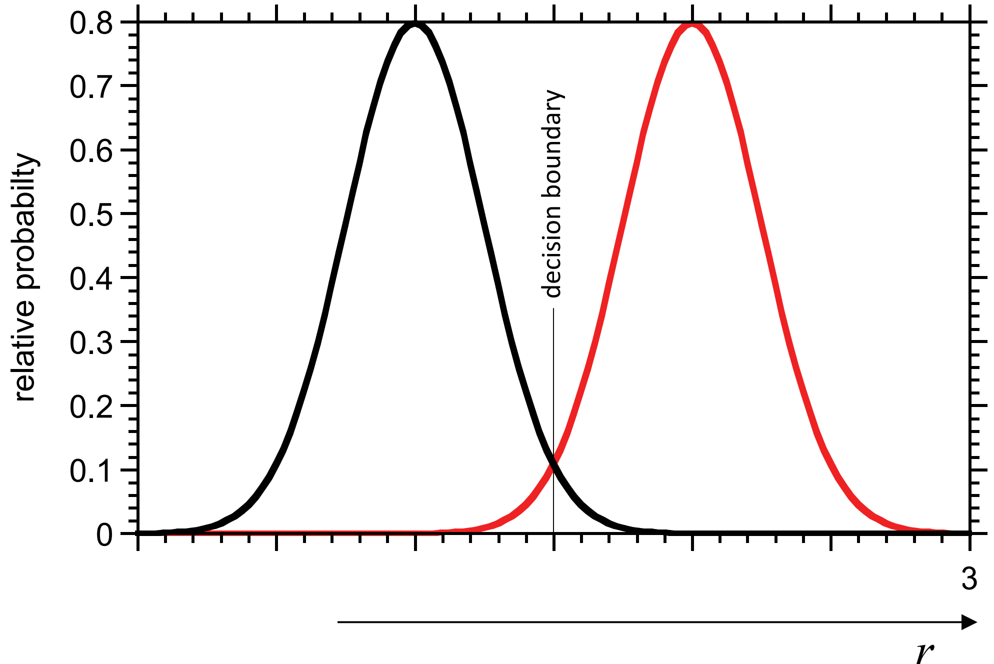
P_{error} = Error probability

= probability of noise crossing the decision boundary

= probability that Gaussian having $\sigma = \sqrt{kTF/2}$ exceeds the value α.

$$P_{error} = Q\left(\frac{\alpha}{\sigma}\right) = Q\left(\frac{\alpha}{\sqrt{kTF/2}}\right)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(\frac{-\beta^2}{2}\right) d\beta$



Error functions

$Q(x)$ can be related to the more well-known error function*

$$Q(x) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right] \quad ,$$

but $Q(x)$ is more directly useful in communications problems.

I will provide a good tabulation of $Q(x)$ for small x,

but there is a very good bound for large x:

$$\left(\frac{x^2}{1+x^2}\right) \cdot \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right) < Q(x) < \frac{1}{x} \cdot \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right)$$

* https://en.wikipedia.org/wiki/Error_function

Tabulated values of the Q-function

Some values of the *Q*-function are given below for reference.

$Q(0.0) = 0.500000000$	$Q(1.0) = 0.158655254$	$Q(2.0) = 0.022750132$	$Q(3.0) = 0.001349898$
$Q(0.1) = 0.460172163$	$Q(1.1) = 0.135666061$	$Q(2.1) = 0.017864421$	$Q(3.1) = 0.000967603$
$Q(0.2) = 0.420740291$	$Q(1.2) = 0.115069670$	$Q(2.2) = 0.013903448$	$Q(3.2) = 0.000687138$
$Q(0.3) = 0.382088578$	$Q(1.3) = 0.096800485$	$Q(2.3) = 0.010724110$	$Q(3.3) = 0.000483424$
$Q(0.4) = 0.344578258$	$Q(1.4) = 0.080756659$	$Q(2.4) = 0.008197536$	$Q(3.4) = 0.000336929$
$Q(0.5) = 0.308537539$	$Q(1.5) = 0.066807201$	$Q(2.5) = 0.006209665$	$Q(3.5) = 0.000232629$
$Q(0.6) = 0.274253118$	$Q(1.6) = 0.054799292$	$Q(2.6) = 0.004661188$	$Q(3.6) = 0.000159109$
$Q(0.7) = 0.241963652$	$Q(1.7) = 0.044565463$	$Q(2.7) = 0.003466974$	$Q(3.7) = 0.000107800$
$Q(0.8) = 0.211855399$	$Q(1.8) = 0.035930319$	$Q(2.8) = 0.002555130$	$Q(3.8) = 0.000072348$
$Q(0.9) = 0.184060125$	$Q(1.9) = 0.028716560$	$Q(2.9) = 0.001865813$	$Q(3.9) = 0.000048096$
			$Q(4.0) = 0.000031671$

http://en.wikipedia.org/wiki/Q-function

Bit error rate vs. SNR: 2-level (binary) modulation

Case 1, binary signals: $m_n = -1, +1$

$$P_{error} = Q\left(\frac{\alpha}{\sigma}\right) = Q\left(\frac{\alpha}{\sqrt{kTF/2}}\right)$$

Received signal energy for one symbol period

$$E_{r,symbol} = \alpha^2 \cdot \left\| m^2 \right\| \cdot (1 \text{ Joule}) = \alpha^2 \text{ because } \left\| m^2 \right\| = 1$$

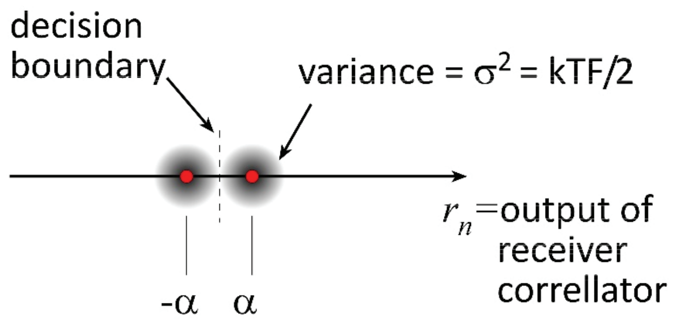
$$P_{error} = Q\left(\sqrt{\frac{E_{r,symbol}}{kTF/2}}\right)$$

where $E_{r,symbol}$ is the average received energy per Symbol

But here: binary, 1 bit/symbol, so $E_{r,symbol} = E_{r,bit}$

where $E_{r,bit}$ is the average received energy per bit

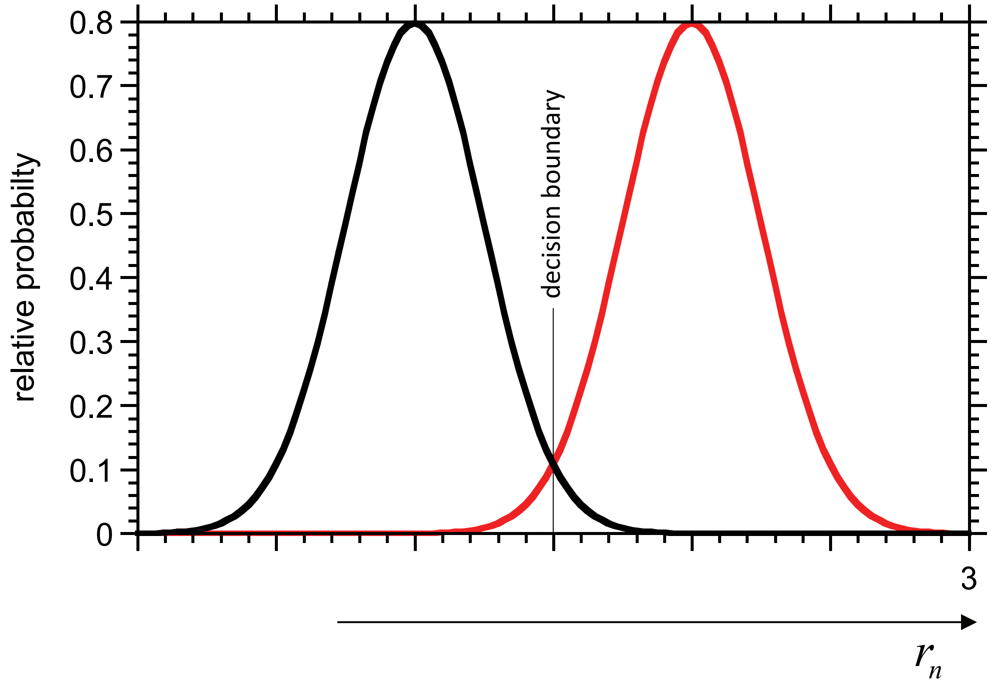
$$P_{error} = Q\left(\sqrt{\frac{E_{r,bit}}{kTF/2}}\right)$$



$m_n = -1$:

Gaussian with mean= $-\alpha$,

standard deviation= $\sigma = \sqrt{kTF/2}$



Bit error rate vs. SNR: 4-level (2bit/symbol) modulation

Case 2, 4-level signals: $m_n = -3, -1, +1, +3$

$$P_{error} = \frac{1}{4}(1+2+2+1)Q\left(\frac{\alpha}{\sigma}\right) = \frac{3}{2}\,Q\left(\frac{\alpha}{\sqrt{kTF/2}}\right)$$

Received signal energy for one symbol period

$$E_{r,symbol} = \alpha^2 \cdot \left\| m^2 \right\| \cdot (1\text{ Joule}) = 5\alpha^2 \text{ because } \left\| m^2 \right\| = 5$$

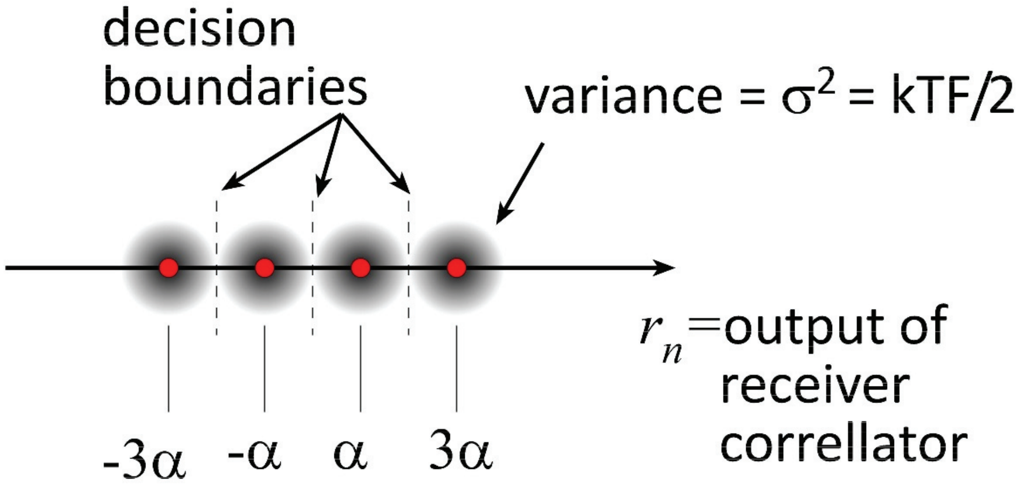
$$P_{error} = \frac{3}{2}\,Q\left(\sqrt{\frac{E_{r,symbol}/5}{kTF/2}}\right)$$

where $E_{r,symbol}$ is the average received energy per Symbol

But here: 4-level, 2 bits/symbol, so $E_{r,symbol} = 2E_{r,bit}$, so

$$P_{error} = \frac{3}{2}\,Q\left(\sqrt{\frac{E_{r,bit}(2/5)}{kTF/2}}\right)$$

where $E_{r,bit}$ is the average received energy per bit



Bit error rate vs. SNR: 8-level (3bit/symbol) modulation

Case 3, 8-level signals: $m_n = -7, -5 - 3, -1, +1, +3, +5, +7$

$$P_{error} = \frac{1}{8}(1+2+2+2+2+2+2+1)Q\left(\frac{\alpha}{\sigma}\right) = \frac{14}{8}\,Q\left(\frac{\alpha}{\sqrt{kTF/2}}\right)$$

Received signal energy for one symbol period

$$E_{r,symbol} = \alpha^2 \cdot \left\| m^2 \right\| \cdot (1\text{ Joule}) = 21\alpha^2 \text{ because } \left\| m^2 \right\| = 21$$

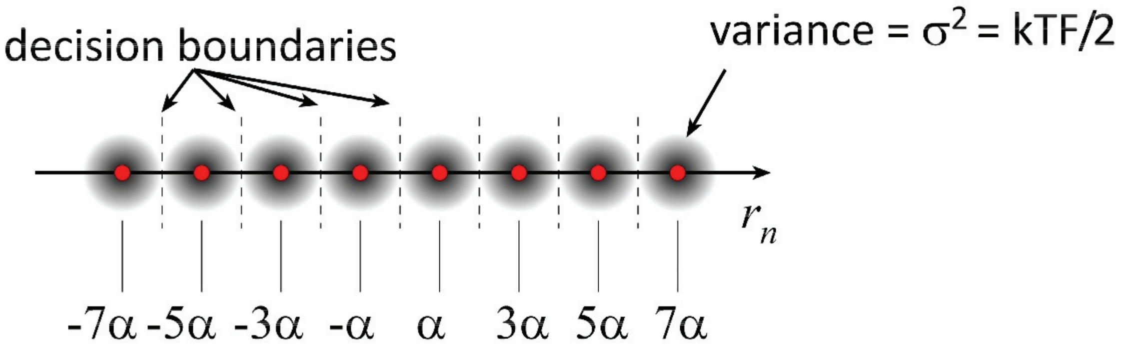
$$P_{error} = \frac{14}{8}\,Q\left(\sqrt{\frac{E_{r,symbol}/21}{kTF/2}}\right)$$

where $E_{r,symbol}$ is the average received energy per Symbol

But here: 4-level, 3 bits/symbol, so $E_{r,symbol} = 3E_{r,bit}$, so

$$P_{error} = \frac{14}{8}\,Q\left(\sqrt{\frac{E_{r,bit}(3/21)}{kTF/2}}\right)$$

where $E_{r,bit}$ is the average received energy per bit



Note that the multi-level coding schemes, though they provide more bits/symbol, hence more bits/Hz, require more energy/bit

The data symbols can include the RF carrier

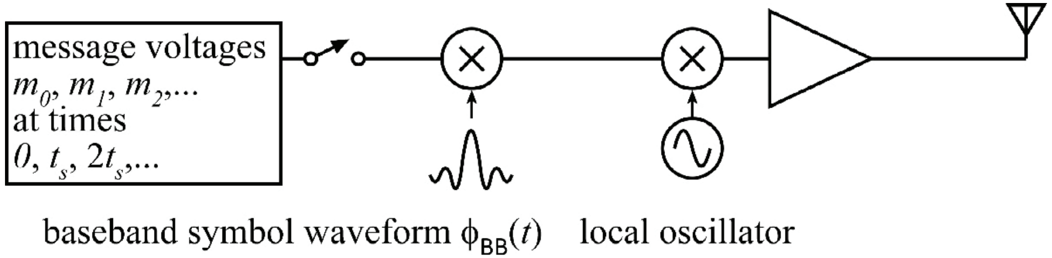
In the prior analysis, the transmitter did not have an LO and RF mixer.

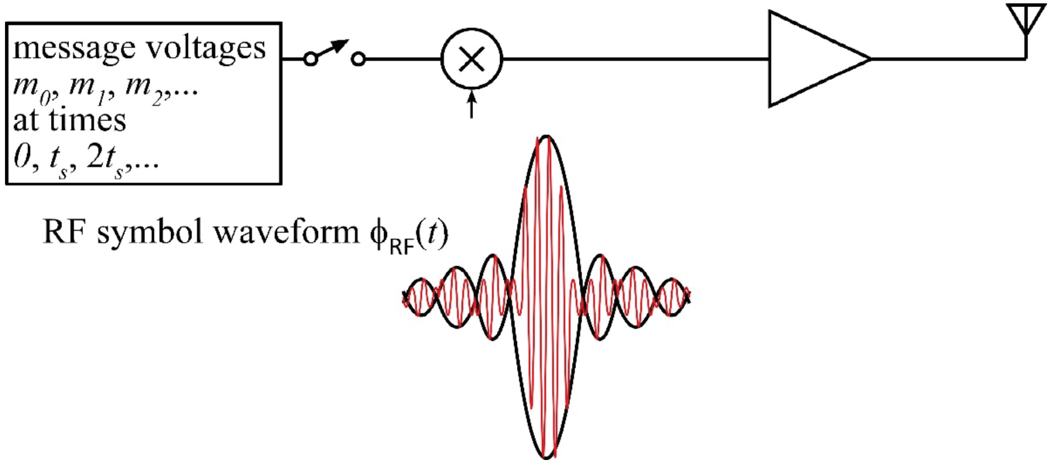
This does not matter: the data symbols can include the RF carrier.

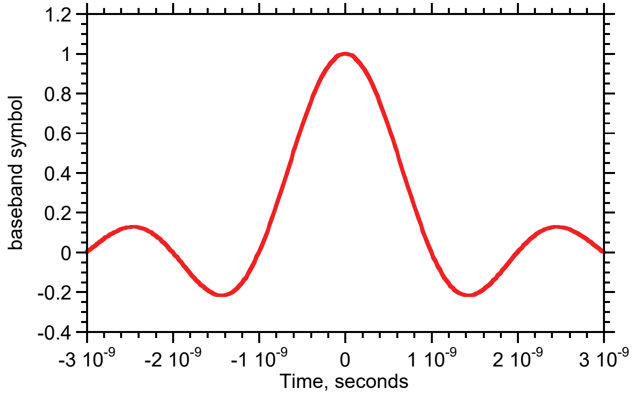
Specifically, if

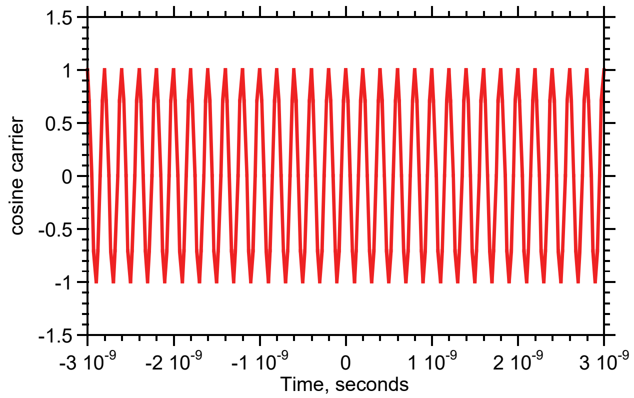
$$\phi_{RF}(t) = \phi_{BB}(t) \cdot 2^{1/2} \cos(\omega_{RF} t)$$

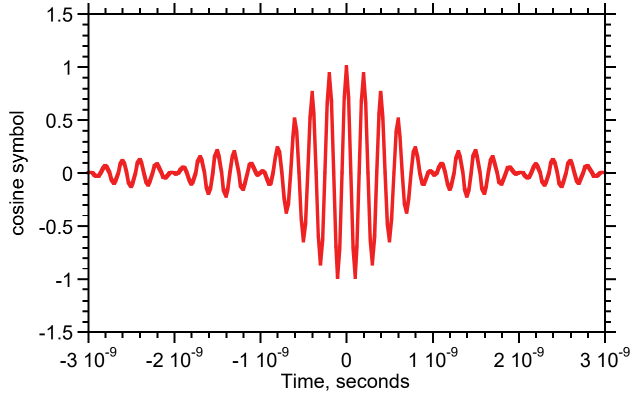
then the 2 block diagrams below are equivalent







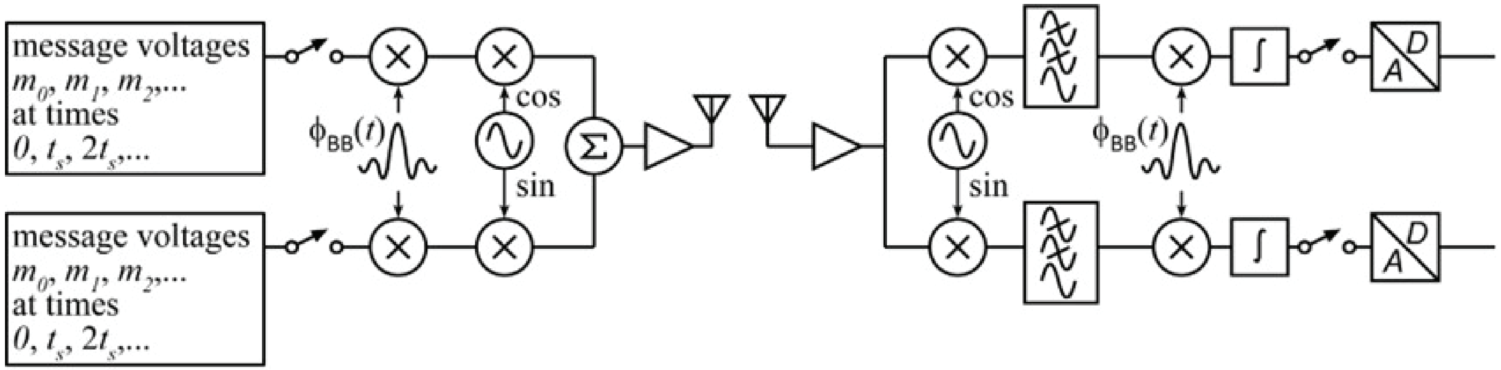


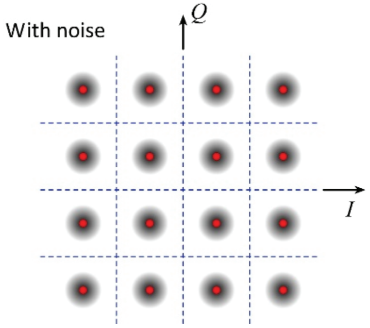


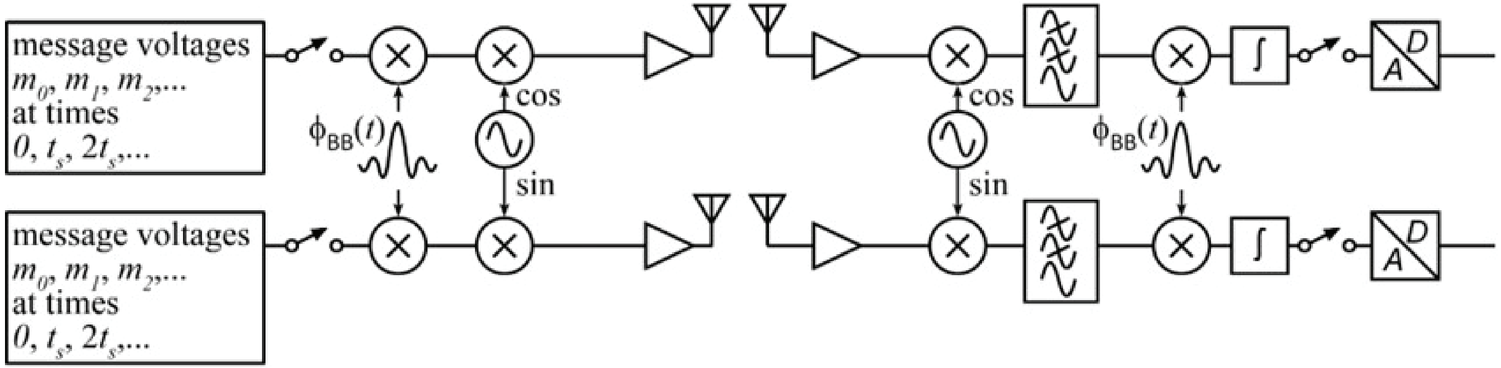
QAM can be treated as two separate radios

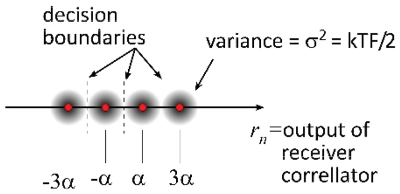
In the prior analysis, we did not consider (I,Q) modulation. But, because $\langle \cos(\omega_{RF}t) | \sin(\omega_{RF}t) \rangle = 0$,

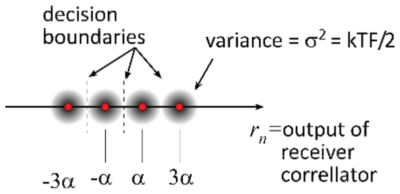
we can, at least neglecting system imperfections, treat the $\cos(\omega_{RF}t)$ and $\sin(\omega_{RF}t)$ signal channels as entirely separate.







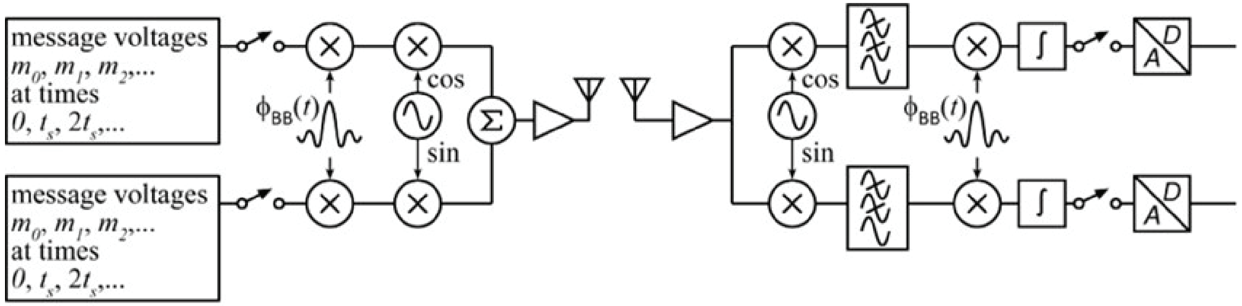




QAM can be treated as two separate radios

We can analyze QAM by separately analyzing the I and Q channels. Consequently,

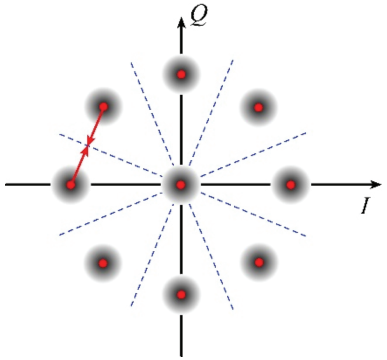
$$P_{error} = \begin{cases} Q\left(\sqrt{\dfrac{E_{r,bit}}{kTF/2}}\right) & \text{QPSK} \\ \dfrac{3}{2}\,Q\left(\sqrt{\dfrac{E_{r,bit}(2/5)}{kTF/2}}\right) & \text{16QAM} \\ P_{error} = \dfrac{14}{8}\,Q\left(\sqrt{\dfrac{E_{r,bit}(3/21)}{kTF/2}}\right) & \text{64QAM} \end{cases}$$

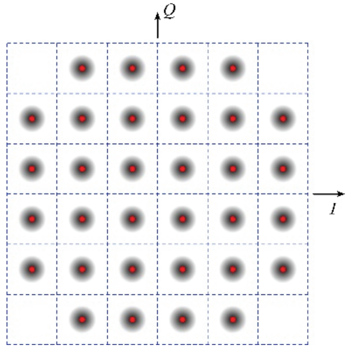


where $E_{r,bit}$ is the average received energy per bit.

If we wish to write expressions in terms of $E_{r,symbol}$, we must decide whether to define this at the total energy per symbol in I and Q, or the energy of each separately.
The first definition is the standard one. Be aware of this factor of 2 if using the $E_{r,symbol}$ expressions from the previous pages.

Decomposing the (I,Q) plane into separated (I) and (Q) analyses is not possible for all constellations....

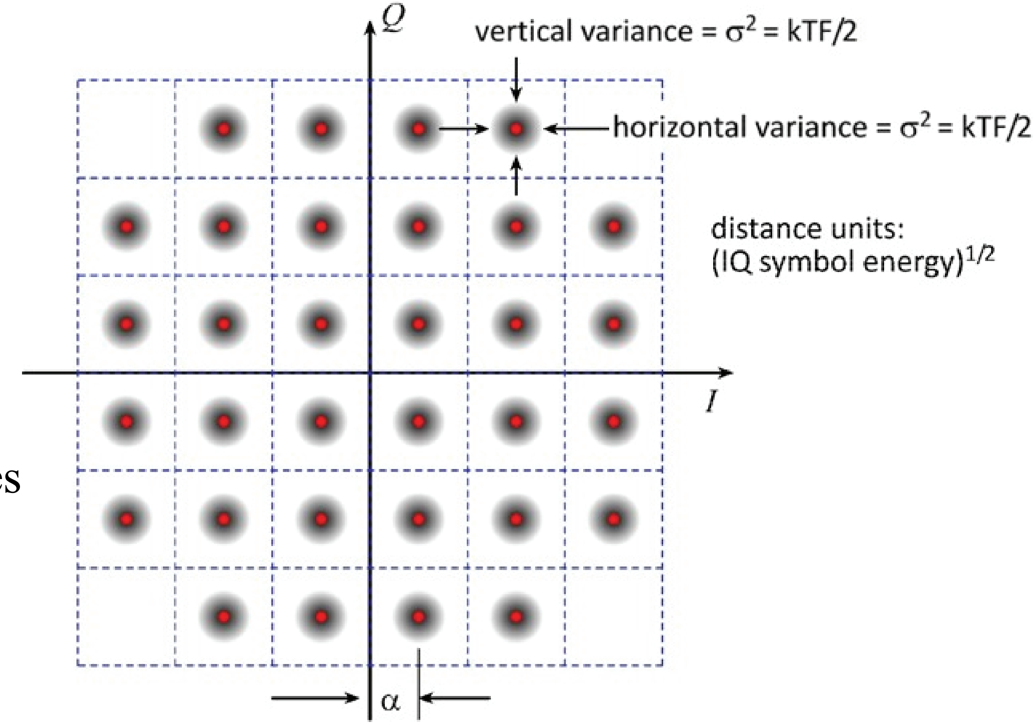




Analysis in the I/Q plane

With an arbitrary (I,Q) consellation, error rates can be found by noting

1) received energy/symbol of each constellation point =(distance from origin)^2



2) Average received energy/symbol =
(energy/symbol of each point) times (probability of each point),
summed over all points.

3) Error probabililty for each contstellation point

$$=\Sigma\, Q\left(\frac{\text{distance to each close boundary}}{\sqrt{kTF/2}}\right) \text{ summed over all close boundaries}$$

4) Overall probability of error per symbol =
(error probabilty for each pont) times (probability of each point),
summed over all points.

3) The relationship between *Bit* error rate and *symbol* error rate
depends on the details of how bits are mapped into symbols

Receivers: correlators vs. matched filters

Output of correlators

$r \propto \langle v_r(t) | \varphi(t) \rangle \propto \int v_r(t)\varphi(t)dt$

What if we instead pass the signal through a matched filter ?

Matched filter impulse response: $h_1(t)$

Output of matched filter: $v_{out}(t) = \int_{-\infty}^{\infty} v_r(\tau)h_1(t-\tau)d\tau$

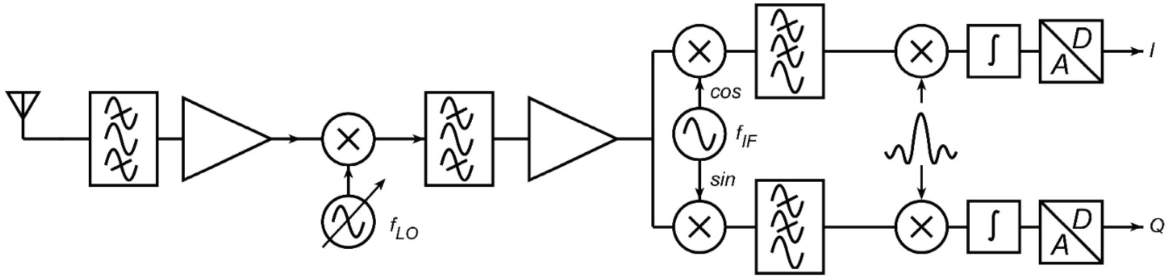
Suppose we set $h_1(t) = \varphi(T-t)$ time reversal and delay.

Then $v_{out}(t) = \int_{-\infty}^{\infty} v_r(\tau)\varphi(T-t+\tau)d\tau$

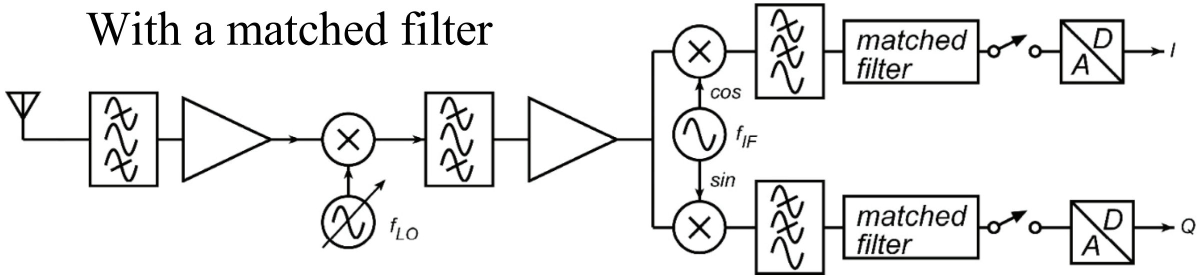
If we sample $v_{out}(t)$ at $t = T$:

$$v_{out}(T) = \int_{-\infty}^{\infty} v_r(\tau)\varphi(\tau)d\tau \propto \langle v_r(t) | \varphi(t) \rangle$$

So we can replace a correlator in the receiver



With a matched filter



Implementation: the root-raised-cosine filters discussed earlier.

These can be analog filters, or can be in DSP after the ADC

Link budget calculations

2015_2_1_link_budget_60GHz.xls [Compatibility Mode] - Excel

FILE HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW VIEW

C27 | 3

Boldface indicates parameters to enter, other parameters are calculated by formula and should be left alone

This spreadsheet calculates power levels for 4QPSK point-point digital microwave radio links along the surface

To calculate RANGE, vary the range until the transmit power (cell F4) is at the appropriate level — 30

	B	C	D	E	F	G	H	I
4	B: Bit rate	**1.25E+10**	1/sec	4QAM required radiated power	-1.0	dBm	8.033E-04	W
5	carrier frequency	**6.00E+10**	Hz	output power per element	-11.0	dBm	7.96E-05	W
6	l: wavelength	5.00E-03	m	PA backoff (Ppeak vs Psat)	**6.0**	dB		
7	Required SNR (measured as Eb/No)	**6.3**	dB	PA saturated output power	-5.0	dBm	3.17E-04	W
8	Receiver bandwidth	**2.16E+09**	Hz	EIRP	30.7	dBm		
9	SNR (measured as kTFB, B from above cell)	14.0	dB	dB EIRP below FCC limits	9.3	dB		
10	F: receiver noise figure	**4.5**	dB	Transmitter				
11	R: transmission range	**50.0**	m	A_effective	2.89E-03	meters^2	115.49	Wavelengths^2
12	atmospheric loss	2.653E-02	dB/m	Vertical beam angle, FWHM	**2.5**	deg	0.0436	radians
13	Dant, trans transmit antenna directivity	1.45E+03	none	Horizontal beam angle, FWHM	**11.3**	deg	0.1972	radians
14	Dant, rcvr receive antenna directivity	1.45E+03	none	array rows and columns	**2**	# rows	**8**	# columns
15	a: bandwidth factor (0.5<a<1)	**0.80**		total # array elements	16			
16	radiated channel bandwidth required	10000.0	MHz	vertical angle scanned, total	5.0	deg		
17				horizontal angle scanned, total	90.4	deg		
18				array height	22.9	wavelengths		
19				array width	5.1	wavelengths		
20				array height	1.15E-01	meters	4.51	inches
21				array width	2.54E-02	meters	1.00	inches
22	kT	-173.83	dBm (1Hz)	Antenna directivity, dB	31.62	dB		
23	packaging loss (receiver)	**2**	dB	Receiver				
24	packaging loss (transmitter)	**2**	dB	A_effective	2.89E-03	meters^2	115.49	Wavelengths^2
25	end-of-life hardware degradation	**3**	dB	Vertical beam angle, FWHM	**2.5**	deg	0.0436	radians
26	hardware design margin	**3**	dB	Horizontal beam angle, FWHM	**11.3**	deg	0.1972	radians
27	beam aiming loss (edge of beam)	**3**	dB	array rows and columns	**2**	# rows	**8**	# columns
28	systems operating margin	**6**	dB	vertical angle scanned, total	5	deg		
29	Prec, received power at 1E-9 BER	-45.03	dBm	horizontal angle scanned, total	90.4	deg		
30	geometric path loss	1.33E-04		array height	2.3E+01	wavelengths		
31	geometric path loss, dB	-38.75	dB	array width	5.1E+00	wavelengths		
32	path obstruction loss (foliage, glass)	**4.00**	dB	array height	1.15E-01	meters	4.51	inches
33	atmospheric loss, dB	1.32655805	dB	array width	2.54E-02	meters	1.00	inches
34	atmospheric loss	26.53	dB/km	Antenna directivity, dB	31.62	dB		

16QAM			
5.6	dBm (peak)	3.61E-03	W
-4.5	dBm (peak)	3.58E-04	W
1.5	dBm	1.43E-03	W
37.2	dBm (peak)	5.25E+00	W
2.8			

64QAM			
10.0	dBm (peak)	1.00E-02	W
0.0	dBm (peak)	9.95E-04	W
		1.00E-03	
6.0	dBm	3.96E-03	W
41.6	dBm (peak)	1.46E+01	W
-1.6			

use 6dB backoff for 16qam

use 3dB backoff for QPSK

rain attenuation fits from Olesn, Rogers, Hodge, IEEE Trans Ant and Prop, March 1978					
Rain rate, mm/hr	**25**	mm/hr		0.98	inch/hr
Ga	4.09E-02		Gb	2.63	
Ea	6.99E-01		Eb	-0.272	
a	7.16E-01		b	8.64E-01	
alpha=aR^b	1.15E+01	dB/km	zero-rain-rate attenuation	15	dB/km
			must read cell E31 from the chart to the right -->		
			0.012 for 10 Ghz, 0.08 for 30 GHz, 5 for 300 GHz		

array height formula is incorrect for angles exceeding about 60 degrees

array width formula is only approximate; more exact analysis of beam width vs array width is on the other spreadsheet page

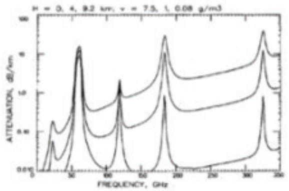


Fig. 2 - Atmospheric attenuation at sea level, 4 kilometers, and 9.2 kilometers elevation (after Liebe).

$$D = 4\pi A_{eff}/\lambda^2 \cong \frac{4\pi}{\theta_{FWHM}^{radians}\phi_{FHWM}^{radians}} \cong \frac{41{,}000}{\theta_{FWHM}^{o}\phi_{FHWM}^{o}}$$

Isotropic antennas: $D = 1, \quad A_{eff} = \lambda^2/4\pi$

$$P_{received}/P_{trans} = (D_t D_r/16\pi^2)(\lambda/R)^2$$

$$P_{received\,(4QPSK)} = Q^2 \cdot kTFB \quad \text{where} \quad Q = \text{SNR}$$

$\lambda/2$ dipoles: $D = 1.64, \quad A_{eff} \approx (\lambda/2)\bullet(\lambda/4)$, $\theta_{FWHM} = 360^0 \quad \phi_{FWHM} = 70^o$

$$\frac{H_{antenna}}{\lambda} \cong \frac{1}{\phi_{FHWM}^{radians}} = \frac{180/\pi}{\phi_{FHWM}^{o}}$$

$$\frac{W_{antenna}}{\lambda} \cong \frac{1}{\theta_{FHWM}^{radians}} = \frac{180/\pi}{\theta_{FHWM}^{o}}$$